CHAPTER 3
PRESSURE AND FLUID STATICS

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3-1C
Solution We are to discuss the difference between gage pressure and absolute pressure.

Analysis The pressure relative to the atmospheric pressure is called the gage pressure, and the pressure relative to an absolute vacuum is called absolute pressure.

Discussion Most pressure gages (like your bicycle tire gage) read relative to atmospheric pressure, and therefore read the gage pressure.

3-2C
Solution We are to explain nose bleeding and shortness of breath at high elevation.

Analysis Atmospheric air pressure which is the external pressure exerted on the skin decreases with increasing elevation. Therefore, the pressure is lower at higher elevations. As a result, the difference between the blood pressure in the veins and the air pressure outside increases. This pressure imbalance may cause some thin-walled veins such as the ones in the nose to burst, causing bleeding. The shortness of breath is caused by the lower air density at higher elevations, and thus lower amount of oxygen per unit volume.

Discussion People who climb high mountains like Mt. Everest suffer other physical problems due to the low pressure.

3-3C
Solution We are to examine a claim about absolute pressure.

Analysis No, the absolute pressure in a liquid of constant density does not double when the depth is doubled. It is the gage pressure that doubles when the depth is doubled.

Discussion This is analogous to temperature scales – when performing analysis using something like the ideal gas law, you must use absolute temperature (K), not relative temperature (°C), or you will run into the same kind of problem.

3-4C
Solution We are to compare the pressure on the surfaces of a cube.

Analysis Since pressure increases with depth, the pressure on the bottom face of the cube is higher than that on the top. The pressure varies linearly along the side faces. However, if the lengths of the sides of the tiny cube suspended in water by a string are very small, the magnitudes of the pressures on all sides of the cube are nearly the same.

Discussion In the limit of an “infinitesimal cube”, we have a fluid particle, with pressure \( P \) defined at a “point”.

3-5C
Solution We are to define Pascal’s law and give an example.

Analysis Pascal’s law states that the pressure applied to a confined fluid increases the pressure throughout by the same amount. This is a consequence of the pressure in a fluid remaining constant in the horizontal direction. An example of Pascal’s principle is the operation of the hydraulic car jack.

Discussion Students may have various answers to the last part of the question. The above discussion applies to fluids at rest (hydrostatics). When fluids are in motion, Pascal’s principle does not necessarily apply. However, as we shall see in later chapters, the differential equations of incompressible fluid flow contain only pressure gradients, and thus an increase in pressure in the whole system does not affect fluid motion.
Chapter 3 Pressure and Fluid Statics

3-6C

**Solution**  We are to compare the volume and mass flow rates of two fans at different elevations.

**Analysis**  The density of air at sea level is higher than the density of air on top of a high mountain. Therefore, the volume flow rates of the two fans running at identical speeds will be the same, but the mass flow rate of the fan at sea level will be higher.

**Discussion**  In reality, the fan blades on the high mountain would experience less frictional drag, and hence the fan motor would not have as much resistance – the rotational speed of the fan on the mountain may be slightly higher than that at sea level.

3-7

**Solution**  A gas is contained in a vertical cylinder with a heavy piston. The pressure inside the cylinder and the effect of volume change on pressure are to be determined.

**Assumptions**  Friction between the piston and the cylinder is negligible.

**Analysis**  

(a) The gas pressure in the piston–cylinder device depends on the atmospheric pressure and the weight of the piston. Drawing the free-body diagram of the piston as shown in Fig. 3–20 and balancing the vertical forces yield

\[ PA = P_{\text{atm}} A + W \]

Solving for \( P \) and substituting,

\[ P = P_{\text{atm}} + \frac{mg}{A} = 95 \text{ kPa} + \frac{(85 \text{ kg})(9.81 \text{ m/s}^2)}{0.04 \text{ m}^2} \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1 \text{ kN/m}^2} \right) = 116 \text{ kPa} \]

(b) The volume change will have no effect on the free-body diagram drawn in part (a), and therefore we do not expect the pressure inside the cylinder to change – it will remain the same.

**Discussion**  If the gas behaves as an ideal gas, the absolute temperature doubles when the volume is doubled at constant pressure.

3-8

**Solution**  The pressure in a vacuum chamber is measured by a vacuum gage. The absolute pressure in the chamber is to be determined.

**Analysis**  The absolute pressure in the chamber is determined from

\[ P_{\text{abs}} = P_{\text{atm}} - P_{\text{vac}} = 92 - 36 = 56 \text{ kPa} \]

**Discussion**  We must remember that “vacuum pressure” is the negative of gage pressure – hence the negative sign.
3-9E

Solution The pressure in a tank is measured with a manometer by measuring the differential height of the manometer fluid. The absolute pressure in the tank is to be determined for two cases: the manometer arm with the \((a)\) higher and \((b)\) lower fluid level being attached to the tank.

Assumptions The fluid in the manometer is incompressible.

Properties The specific gravity of the fluid is given to be \(SG = 1.25\). The density of water at \(32^\circ F\) is \(62.4 \text{ lbm/ft}^3\).

Analysis The density of the fluid is obtained by multiplying its specific gravity by the density of water,

\[
\rho = SG \times \rho_{\text{water}} = (1.25)(62.4 \text{ lbm/ft}^3) = 78.0 \text{ lbm/ft}^3
\]

The pressure difference corresponding to a differential height of 28 in between the two arms of the manometer is

\[
\Delta P = \rho gh = (78 \text{ lbm/ft}^3)(32.174 \text{ ft/s}^2)(28/12 \text{ ft})\left(\frac{1 \text{ lb}}{32.174 \text{ lbm-ft/s}^2} \right)\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) = 1.26 \text{ psia}
\]

Then the absolute pressures in the tank for the two cases become:

\(a\) The fluid level in the arm attached to the tank is higher (vacuum):

\[
P_{\text{abs}} = P_{\text{atm}} - P_{\text{vac}} = 12.7 - 1.26 = 11.44 \text{ psia} \approx 11.4 \text{ psia}
\]

\(b\) The fluid level in the arm attached to the tank is lower:

\[
P_{\text{abs}} = P_{\text{gage}} + P_{\text{atm}} = 12.7 + 1.26 = 13.96 \text{ psia} \approx 14.0 \text{ psia}
\]

Discussion The final results are reported to three significant digits. Note that we can determine whether the pressure in a tank is above or below atmospheric pressure by simply observing the side of the manometer arm with the higher fluid level.
3-10
Solution  The pressure in a pressurized water tank is measured by a multi-fluid manometer. The gage pressure of air in the tank is to be determined.

Assumptions  The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus we can determine the pressure at the air-water interface.

Properties  The densities of mercury, water, and oil are given to be 13,600, 1000, and 850 kg/m³, respectively.

Analysis  Starting with the pressure at point 1 at the air-water interface, and moving along the tube by adding (as we go down) or subtracting (as we go up) the $\rho gh$ terms until we reach point 2, and setting the result equal to $P_{\text{atm}}$ since the tube is open to the atmosphere gives

$$P_1 + \rho_{\text{water}} gh_1 + \rho_{\text{oil}} gh_2 - \rho_{\text{mercury}} gh_3 = P_{\text{atm}}$$

Solving for $P_1$,

$$P_1 = P_{\text{atm}} - \rho_{\text{water}} gh_1 - \rho_{\text{oil}} gh_2 + \rho_{\text{mercury}} gh_3$$

or,

$$P_1 - P_{\text{atm}} = g(\rho_{\text{mercury}} h_3 - \rho_{\text{water}} h_1 - \rho_{\text{oil}} h_2)$$

Noting that $P_{1,\text{gage}} = P_1 - P_{\text{atm}}$ and substituting,

$$P_{1,\text{gage}} = (9.81 \text{ m/s}^2) [(13,600 \text{ kg/m}^3)(0.46 \text{ m}) - (1000 \text{ kg/m}^3)(0.2 \text{ m})$$

$$- (850 \text{ kg/m}^3)(0.3 \text{ m})] \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right)$$

$$= 56.9 \text{ kPa}$$

Discussion  Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly.

3-11
Solution  The barometric reading at a location is given in height of mercury column. The atmospheric pressure is to be determined.

Properties  The density of mercury is given to be 13,600 kg/m³.

Analysis  The atmospheric pressure is determined directly from

$$P_{\text{atm}} = \rho gh$$

$$= (13,600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.735 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right)$$

$$= 98.1 \text{ kPa}$$

Discussion  We round off the final answer to three significant digits. 100 kPa is a fairly typical value of atmospheric pressure on land slightly above sea level.
3-12
Solution  The gage pressure in a liquid at a certain depth is given. The gage pressure in the same liquid at a different depth is to be determined.

Assumptions  The variation of the density of the liquid with depth is negligible.

Analysis  The gage pressure at two different depths of a liquid can be expressed as  \( P_1 = \rho gh_1 \) and  \( P_2 = \rho gh_2 \). Taking their ratio,

\[
\frac{P_2}{P_1} = \frac{\rho gh_2}{\rho gh_1} = \frac{h_2}{h_1}
\]

Solving for  \( P_2 \) and substituting gives

\[
P_2 = \frac{h_2}{h_1} P_1 = \frac{12 \text{ m}}{3 \text{ m}} (28 \text{ kPa}) = 112 \text{ kPa}
\]

Discussion  Note that the gage pressure in a given fluid is proportional to depth.

3-13
Solution  The absolute pressure in water at a specified depth is given. The local atmospheric pressure and the absolute pressure at the same depth in a different liquid are to be determined.

Assumptions  The liquid and water are incompressible.

Properties  The specific gravity of the fluid is given to be \( SG = 0.78 \). We take the density of water to be 1000 kg/m\(^3\). Then density of the liquid is obtained by multiplying its specific gravity by the density of water,

\[
\rho = SG \times \rho_{H_2O} = (0.78)(1000 \text{ kg/m}^3) = 780 \text{ kg/m}^3
\]

Analysis  

(a) Knowing the absolute pressure, the atmospheric pressure can be determined from

\[
P_{\text{atm}} = P - \rho gh
\]

\[
= (145 \text{ kPa}) - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5 \text{ m}) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) = 96.0 \text{ kPa}
\]

(b) The absolute pressure at a depth of 5 m in the other liquid is

\[
P = P_{\text{atm}} + \rho gh
\]

\[
= (96.0 \text{ kPa}) + (780 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5 \text{ m}) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right)
\]

\[
= 134.3 \text{ kPa} \approx 134 \text{ kPa}
\]

Discussion  Note that at a given depth, the pressure in the lighter fluid is lower, as expected.

3-14E
Solution  It is to be shown that 1 kgf/cm\(^2\) = 14.223 psi.

Analysis  Noting that 1 kgf = 9.80665 N, 1 N = 0.22481 lbf, and 1 in = 2.54 cm, we have

\[
1 \text{ kgf} = 9.80665 \text{ N} = (9.80665 \text{ N}) \left( \frac{0.22481 \text{ lbf}}{1 \text{ N}} \right) = 2.20463 \text{ lbf}
\]

and

\[
1 \text{ kgf/cm}^2 = 2.20463 \text{ lbf/cm}^2 = (2.20463 \text{ lbf/cm}^2) \left( \frac{2.54 \text{ cm}}{1 \text{ in}} \right)^2 = 14.223 \text{ lbf/in}^2 = 14.223 \text{ psi}
\]

Discussion  This relationship may be used as a conversion factor.
3-15E
Solution  The weight and the foot imprint area of a person are given. The pressures this man exerts on the ground when he stands on one and on both feet are to be determined.

Assumptions  The weight of the person is distributed uniformly on foot imprint area.

Analysis  The weight of the man is given to be 200 lbf. Noting that pressure is force per unit area, the pressure this man exerts on the ground is

(a) On one foot: \[ P = \frac{W}{A} = \frac{200 \text{ lbf}}{36 \text{ in}^2} = 5.56 \text{ lbf/in}^2 = 5.56 \text{ psi} \]

(b) On both feet: \[ P = \frac{W}{2A} = \frac{200 \text{ lbf}}{2 \times 36 \text{ in}^2} = 2.78 \text{ lbf/in}^2 = 2.78 \text{ psi} \]

Discussion  Note that the pressure exerted on the ground (and on the feet) is reduced by half when the person stands on both feet.

3-16
Solution  The mass of a woman is given. The minimum imprint area per shoe needed to enable her to walk on the snow without sinking is to be determined.

Assumptions  1 The weight of the person is distributed uniformly on the imprint area of the shoes. 2 One foot carries the entire weight of a person during walking, and the shoe is sized for walking conditions (rather than standing). 3 The weight of the shoes is negligible.

Analysis  The mass of the woman is given to be 55 kg. For a pressure of 0.5 kPa on the snow, the imprint area of one shoe must be

\[ A = \frac{W}{P} = \frac{mg}{P} = \frac{(55 \text{ kg})(9.81 \text{ m/s}^2)}{0.5 \text{ kPa}} = \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) = 1.08 \text{ m}^2 \]

Discussion  This is a very large area for a shoe, and such shoes would be impractical to use. Therefore, some sinking of the snow should be allowed to have shoes of reasonable size.

3-17
Solution  The vacuum pressure reading of a tank is given. The absolute pressure in the tank is to be determined.

Properties  The density of mercury is given to be \( \rho = 13,590 \text{ kg/m}^3 \).

Analysis  The atmospheric (or barometric) pressure can be expressed as

\[ P_{\text{atm}} = \rho gh \]

\[ = (13,590 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(0.755 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \]

\[ = 100.6 \text{ kPa} \]

Then the absolute pressure in the tank becomes

\[ P_{\text{abs}} = P_{\text{atm}} - P_{\text{vac}} = 100.6 - 30 = 70.6 \text{ kPa} \]

Discussion  The gage pressure in the tank is the negative of the vacuum pressure, i.e., \( P_{\text{gage}} = -30.0 \text{ kPa} \).
3-18E
Solution A pressure gage connected to a tank reads 50 psi. The absolute pressure in the tank is to be determined.

Properties The density of mercury is given to be \( \rho = 848.4 \text{ lbm/ft}^3 \).

Analysis The atmospheric (or barometric) pressure can be expressed as

\[
P_{\text{atm}} = \rho gh = (848.4 \text{ lbm/ft}^3)(32.174 \text{ ft/s}^2)(29.1/12 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.174 \text{ lbm-ft/s}^2} \right) \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right)
\]

Then the absolute pressure in the tank is

\[
P_{\text{abs}} = P_{\text{gage}} + P_{\text{atm}} = 50 + 14.29 \approx 64.3 \text{ psia}
\]

Discussion This pressure is more than four times as much as standard atmospheric pressure.

3-19
Solution A pressure gage connected to a tank reads 500 kPa. The absolute pressure in the tank is to be determined.

Analysis The absolute pressure in the tank is determined from

\[
P_{\text{abs}} = P_{\text{gage}} + P_{\text{atm}} = 500 + 94 = 594 \text{ kPa}
\]

Discussion This pressure is almost six times greater than standard atmospheric pressure.
Water is raised from a reservoir through a vertical tube by the sucking action of a piston. The force needed to raise the water to a specified height is to be determined, and the pressure at the piston face is to be plotted against height.

**Assumptions** 1 Friction between the piston and the cylinder is negligible. 2 Accelerational effects are negligible.

**Properties** We take the density of water to be $\rho = 1000 \text{ kg/m}^3$.

**Analysis** Noting that the pressure at the free surface is $P_{\text{atm}}$ and hydrostatic pressure in a fluid decreases linearly with increasing height, the pressure at the piston face is

$$P = P_{\text{atm}} - \rho gh = 95 \text{ kPa} - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.5 \text{ m}) \left( \frac{1 \text{kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{kPa}}{1 \text{kN/m}^2} \right) = 81.3 \text{ kPa}$$

Piston face area is

$$A = \pi D^2 / 4 = \pi (0.3 \text{ m})^2 / 4 = 0.07069 \text{ m}^2$$

A force balance on the piston yields

$$F = (P_{\text{atm}} - P)A = (95 - 81.3 \text{ kPa})(0.07068 \text{ m}^2) \left( \frac{1 \text{kN/m}^2}{1 \text{kPa}} \right) \left( \frac{1 \text{kPa}}{1 \text{kN/m}^2} \right) = 1.04 \text{ kN}$$

Repeating calculations for $h = 3 \text{ m}$ gives $P = 66.6 \text{ kPa}$ and $F = 2.08 \text{ kN}$.

Using EES, the absolute pressure can be calculated from $P = P_{\text{atm}} - \rho gh$ for various values of $h$ from 0 to 3 m, and the results can be plotted as shown below:

**Discussion** Note that the pressure at the piston face decreases, and the force needed to raise water increases linearly with increasing height of water column relative to the free surface.
3-21

**Solution**  A mountain hiker records the barometric reading before and after a hiking trip. The vertical distance climbed is to be determined.

**Assumptions**  The variation of air density and the gravitational acceleration with altitude is negligible.

**Properties**  The density of air is given to be $\rho = 1.20 \text{ kg/m}^3$.

**Analysis**  Taking an air column between the top and the bottom of the mountain and writing a force balance per unit base area, we obtain

$$W_{\text{air}}/A = P_{\text{bottom}} - P_{\text{top}} \rightarrow \ (\rho g h)_{\text{air}} = P_{\text{bottom}} - P_{\text{top}} \rightarrow \ h = \frac{P_{\text{bottom}} - P_{\text{top}}}{\rho g}$$

$$h = \frac{(0.930 - 0.820) \text{ bar}}{(1.20 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{100,000 \text{ N/m}^2}{1 \text{ bar}} \right) \left( \frac{1 \text{ kg/m}^s^2}{1 \text{ N}} \right) = 934.42 \text{ m}$$

which to three significant digits is $h = 934 \text{ m}$, which is also the distance climbed.

**Discussion**  A similar principle is used in some aircraft instruments to measure elevation.

3-22

**Solution**  A barometer is used to measure the height of a building by recording reading at the bottom and at the top of the building. The height of the building is to be determined.

**Assumptions**  The variation of air density with altitude is negligible.

**Properties**  The density of air is given to be $\rho = 1.18 \text{ kg/m}^3$. The density of mercury is 13,600 kg/m$^3$.

**Analysis**  Atmospheric pressures at the top and at the bottom of the building are

$$P_{\text{top}} = (\rho g h)_{\text{top}} = (13,600 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(0.730 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) = 97.36 \text{ kPa}$$

$$P_{\text{bottom}} = (\rho g h)_{\text{bottom}} = (13,600 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(0.755 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) = 100.70 \text{ kPa}$$

Taking an air column between the top and the bottom of the building, we write a force balance per unit base area,

$$W_{\text{air}}/A = P_{\text{bottom}} - P_{\text{top}} \quad \text{and} \quad (\rho g h)_{\text{air}} = P_{\text{bottom}} - P_{\text{top}}$$

$$(1.18 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(h) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) = (100.70 - 97.36) \text{ kPa}$$

which yields $h = 288.6 \text{ m} \cong 289 \text{ m}$, which is also the height of the building.

**Discussion**  There are more accurate ways to measure the height of a building, but this method is quite simple.
Solution The previous problem is reconsidered. The EES solution is to be printed out, including proper units.

Analysis The EES Equations window is printed below, followed by the Solution window.

\[
\begin{align*}
P_{\text{bottom}} &= 755 \text{[mmHg]} \\
P_{\text{top}} &= 730 \text{[mmHg]} \\
g &= 9.807 \text{[m/s}^2]\text{]} \quad \text{"local acceleration of gravity at sea level"} \\
rho &= 1.18 \text{[kg/m}^3]\text{]} \\
\Delta P_{\text{abs}} &= (P_{\text{bottom}} - P_{\text{top}}) \times \text{CONVERT('mmHg','kPa')} \text{[kPa]} \quad \text{"Delta P reading from the barometers, converted from mmHg to kPa."} \\
\Delta P_{\text{h}} &= \rho g h/1000 \text{[kPa]} \quad \text{"Equ. 1-16. Delta P due to the air fluid column height, h, between the top and bottom of the building."} \\
\text{Instead of dividing by 1000 Pa/kPa we could have multiplied \( \rho g h \) by the EES function, CONVERT('Pa','kPa')"} \\
\Delta P_{\text{abs}} &= \Delta P_{\text{h}}
\end{align*}
\]

SOLUTION

Variables in Main

\[
\begin{align*}
\Delta P_{\text{abs}} &= 3.333 \text{[kPa]} & \Delta P_{\text{h}} &= 3.333 \text{[kPa]} \\
g &= 9.807 \text{[m/s}^2]\text{]} & h &= 288 \text{[m]} \\
P_{\text{bottom}} &= 755 \text{[mmHg]} & P_{\text{top}} &= 730 \text{[mmHg]} \\
\rho &= 1.18 \text{[kg/m}^3]\text{]} 
\end{align*}
\]

Discussion To obtain the solution in EES, simply click on the icon that looks like a calculator, or Calculate-Solve.

3-24

Solution A diver is moving at a specified depth from the water surface. The pressure exerted on the surface of the diver by the water is to be determined.

Assumptions The variation of the density of water with depth is negligible.

Properties The specific gravity of sea water is given to be \( \text{SG} = 1.03 \). We take the density of water to be 1000 kg/m\(^3\).

Analysis The density of the sea water is obtained by multiplying its specific gravity by the density of water which is taken to be 1000 kg/m\(^3\):

\[
\rho = \text{SG} \times \rho_{\text{H}_2\text{O}} = (1.03)(1000 \text{ kg/m}^3) = 1030 \text{ kg/m}^3
\]

The pressure exerted on a diver at 20 m below the free surface of the sea is the absolute pressure at that location:

\[
P = P_{\text{atm}} + \rho gh
\]

\[
= (101 \text{ kPa}) + (1030 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(20 \text{ m}) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right)
\]

\[
= 303 \text{ kPa}
\]

Discussion This is about 3 times the normal sea level value of atmospheric pressure.
Chapter 3 Pressure and Fluid Statics

3-25E

Solution  A submarine is cruising at a specified depth from the water surface. The pressure exerted on the surface of the submarine by water is to be determined.

Assumptions  The variation of the density of water with depth is negligible.

Properties  The specific gravity of sea water is given to be SG = 1.03. The density of water at 32°F is 62.4 lbm/ft³.

Analysis  The density of the seawater is obtained by multiplying its specific gravity by the density of water,

\[ \rho = \text{SG} \times \rho_{H_2O} = (1.03)(62.4 \text{ lbm/ft}^3) = 64.27 \text{ lbm/ft}^3 \]

The pressure exerted on the surface of the submarine cruising 300 ft below the free surface of the sea is the absolute pressure at that location:

\[ P = P_{atm} + \rho gh \]

\[ = (14.7 \text{ psia}) + (64.27 \text{ lbm/ft}^3)(32.174 \text{ ft/s}^2)(300 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.174 \text{ lbm} \cdot \text{ft/s}^2} \right) \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) \]

\[ = 148.6 \text{ psia} \approx 149 \text{ psia} \]

where we have rounded the final answer to three significant digits.

Discussion  This is more than 10 times the value of atmospheric pressure at sea level.

3-26

Solution  A gas contained in a vertical piston-cylinder device is pressurized by a spring and by the weight of the piston. The pressure of the gas is to be determined.

Analysis  Drawing the free body diagram of the piston and balancing the vertical forces yields

\[ PA = P_{atm} A + W + F_{spring} \]

Thus,

\[ P = P_{atm} + \frac{mg + F_{spring}}{A} \]

\[ = (95 \text{ kPa}) + \frac{(4 \text{ kg})(9.807 \text{ m/s}^2)}{35 \times 10^{-4} \text{ m}^2} + 60 \text{ N} \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) = 123.4 \text{ kPa} \approx 123 \text{ kPa} \]

Discussion  This setup represents a crude but functional way to control the pressure in a tank.
Solution  The previous problem is reconsidered. The effect of the spring force in the range of 0 to 500 N on the pressure inside the cylinder is to be investigated. The pressure against the spring force is to be plotted, and results are to be discussed.

Analysis  The EES Equations window is printed below, followed by the tabulated and plotted results.

\[
g = 9.807 \text{[m/s}^2]\]
\[
P_{\text{atm}} = 95 \text{[kPa]}
\]
\[
m_{\text{piston}} = 4 \text{[kg]}
\]
\[
\{F_{\text{spring}} = 60 \text{[N]}\}
\]
\[
A = 35 \times \text{CONVERT('cm}^2', 'm}^2) \text{[m}^2]\]
\[
W_{\text{piston}} = m_{\text{piston}} g \text{[N]}
\]
\[
F_{\text{atm}} = P_{\text{atm}} A \times \text{CONVERT('kPa', 'N/m}^2) \text{[N]}
\]

"From the free body diagram of the piston, the balancing vertical forces yield:"
\[
F_{\text{gas}} = F_{\text{atm}} + F_{\text{spring}} + W_{\text{piston}} \text{[N]}
\]
\[
P_{\text{gas}} = F_{\text{gas}} / A \times \text{CONVERT('N/m}^2', 'kPa) \text{[kPa]}
\]

Results:

<table>
<thead>
<tr>
<th>(F_{\text{spring}} \text{[N]})</th>
<th>(P_{\text{gas}} \text{[kPa]})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>106.2</td>
</tr>
<tr>
<td>55.56</td>
<td>122.1</td>
</tr>
<tr>
<td>111.1</td>
<td>138</td>
</tr>
<tr>
<td>166.7</td>
<td>153.8</td>
</tr>
<tr>
<td>222.2</td>
<td>169.7</td>
</tr>
<tr>
<td>277.8</td>
<td>185.6</td>
</tr>
<tr>
<td>333.3</td>
<td>201.4</td>
</tr>
<tr>
<td>388.9</td>
<td>217.3</td>
</tr>
<tr>
<td>444.4</td>
<td>233.2</td>
</tr>
<tr>
<td>500</td>
<td>249.1</td>
</tr>
</tbody>
</table>

Discussion  The relationship is linear, as expected.
3.28 [Also solved using EES on enclosed DVD]

**Solution**  Both a pressure gage and a manometer are attached to a tank of gas to measure its pressure. For a specified reading of gage pressure, the difference between the fluid levels of the two arms of the manometer is to be determined for mercury and water.

**Properties**  The densities of water and mercury are given to be $\rho_{\text{water}} = 1000 \text{ kg/m}^3$ and $\rho_{\text{Hg}} = 13,600 \text{ kg/m}^3$.

**Analysis**  The gage pressure is related to the vertical distance $h$ between the two fluid levels by

$$P_{\text{gage}} = \rho g h \quad \longrightarrow \quad h = \frac{P_{\text{gage}}}{\rho g}$$

(a) For mercury,

$$h = \frac{P_{\text{gage}}}{\rho_{\text{Hg}} g} = \frac{65 \text{ kPa}}{(13600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kN/m}^2}{1 \text{ kPa}} \right) \left( \frac{1000 \text{ kg/m} \cdot \text{s}^2}{1 \text{ kN}} \right) = 0.49 \text{ m}$$

(b) For water,

$$h = \frac{P_{\text{gage}}}{\rho_{\text{H,O}} g} = \frac{65 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kN/m}^2}{1 \text{ kPa}} \right) \left( \frac{1000 \text{ kg/m} \cdot \text{s}^2}{1 \text{ kN}} \right) = 6.63 \text{ m}$$

**Discussion**  The manometer with water is more precise since the column height is bigger (better resolution). However, a column of water more than 8 meters high would be impractical, so mercury is the better choice of manometer fluid here. 

*Note:* Mercury vapors are hazardous, and the use of mercury is no longer encouraged.
Solution  The previous problem is reconsidered. The effect of the manometer fluid density in the range of 800 to 13,000 kg/m³ on the differential fluid height of the manometer is to be investigated. Differential fluid height is to be plotted as a function of the density, and the results are to be discussed.

Analysis  The EES Equations window is printed below, followed by the tabulated and plotted results.

Function fluid_density(Fluid$)
  If fluid$='Mercury' then fluid_density=13600 else fluid_density=1000
end

{Input from the diagram window. If the diagram window is hidden, then all of the input must come from the equations window. Also note that brackets can also denote comments - but these comments do not appear in the formatted equations window.}

{Fluid$='Mercury'
P_atm = 101.325                           "kpa"
DELTAP=80             "kPa  Note how DELTAP is displayed on the Formatted Equations Window."}  
g=9.807                                           "m/s², local acceleration of gravity at sea level"
rho=Fluid_density(Fluid$)   "Get the fluid density, either Hg or H2O, from the function"
"To plot fluid height against density place {} around the above equation. Then set up the parametric table and solve."
DELTAP = RHO*g*h/1000
"Instead of dividing by 1000 Pa/kPa we could have multiplied by the EES function, CONVERT('Pa','kPa')"
h_mm=h*convert('m','mm')   "The fluid height in mm is found using the built-in CONVERT function."
P_abs= P_atm + DELTAP

"To make the graph, hide the diagram window and remove the {} brackets from Fluid$ and from P_atm. Select New Parametric Table from the Tables menu. Choose P_abs, DELTAP and h to be in the table. Choose Alter Values from the Tables menu. Set values of h to range from 0 to 1 in steps of 0.2. Choose Solve Table (or press F3) from the Calculate menu. Choose New Plot Window from the Plot menu. Choose to plot P_abs vs h and then choose Overlay Plot from the Plot menu and plot DELTAP on the same scale."

Results:

<table>
<thead>
<tr>
<th>h_mm [mm]</th>
<th>ρ [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10197</td>
<td>800</td>
</tr>
<tr>
<td>3784</td>
<td>2156</td>
</tr>
<tr>
<td>2323</td>
<td>3511</td>
</tr>
<tr>
<td>1676</td>
<td>4867</td>
</tr>
<tr>
<td>1311</td>
<td>6222</td>
</tr>
<tr>
<td>1076</td>
<td>7578</td>
</tr>
<tr>
<td>913.1</td>
<td>8933</td>
</tr>
<tr>
<td>792.8</td>
<td>10289</td>
</tr>
<tr>
<td>700.5</td>
<td>11644</td>
</tr>
<tr>
<td>627.5</td>
<td>13000</td>
</tr>
</tbody>
</table>

Tank Fluid Gage and Absolute Pressures vs Manometer Fluid Height

[Graph showing the relationship between Manometer Fluid Height and Tank Fluid Gage and Absolute Pressures]
Discussion

Many comments are provided in the Equation window above to help you learn some of the features of EES.
3-30
Solution The air pressure in a tank is measured by an oil manometer. For a given oil-level difference between the two columns, the absolute pressure in the tank is to be determined.

Properties The density of oil is given to be $\rho = 850 \text{ kg/m}^3$.

Analysis The absolute pressure in the tank is determined from

$$P = P_{\text{atm}} + \rho gh$$

$$= (98 \text{ kPa}) + (850 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.45 \text{ m})$$

$$= 101.75 \text{ kPa} \approx 102 \text{ kPa}$$

Discussion If a heavier liquid, such as water, were used for the manometer fluid, the column height would be smaller, and thus the reading would be less precise (lower resolution).

3-31
Solution The air pressure in a duct is measured by a mercury manometer. For a given mercury-level difference between the two columns, the absolute pressure in the duct is to be determined.

Properties The density of mercury is given to be $\rho = 13,600 \text{ kg/m}^3$.

Analysis (a) The pressure in the duct is above atmospheric pressure since the fluid column on the duct side is at a lower level.

(b) The absolute pressure in the duct is determined from

$$P = P_{\text{atm}} + \rho gh$$

$$= (100 \text{ kPa}) + (13,600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.010 \text{ m})$$

$$= 101.3 \text{ kPa}$$

Discussion When measuring pressures in a fluid flow, the difference between two pressures is usually desired. In this case, the difference is between the measurement point and atmospheric pressure.

3-32
Solution The air pressure in a duct is measured by a mercury manometer. For a given mercury-level difference between the two columns, the absolute pressure in the duct is to be determined.

Properties The density of mercury is given to be $\rho = 13,600 \text{ kg/m}^3$.

Analysis (a) The pressure in the duct is above atmospheric pressure since the fluid column on the duct side is at a lower level.

(b) The absolute pressure in the duct is determined from

$$P = P_{\text{atm}} + \rho gh$$

$$= (100 \text{ kPa}) + (13,600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.030 \text{ m})$$

$$= 104.00 \text{ kPa} \approx 104 \text{ kPa}$$

Discussion The final result is given to three significant digits.
3-33

**Solution** The systolic and diastolic pressures of a healthy person are given in mm of Hg. These pressures are to be expressed in kPa, psi, and meters of water column.

**Assumptions** Both mercury and water are incompressible substances.

**Properties** We take the densities of water and mercury to be 1000 kg/m³ and 13,600 kg/m³, respectively.

**Analysis** Using the relation \( P = \rho gh \) for gage pressure, the high and low pressures are expressed as

\[
P_{\text{high}} = \rho_{\text{high}} gh_{\text{high}} = (13,600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.12 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) = 16.0 \text{ kPa}
\]

\[
P_{\text{low}} = \rho_{\text{low}} gh_{\text{low}} = (13,600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.08 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) = 10.7 \text{ kPa}
\]

Noting that 1 psi = 6.895 kPa,

\[
P_{\text{high}} = (16.0 \text{ kPa}) \left( \frac{1 \text{ psi}}{6.895 \text{kPa}} \right) = 2.32 \text{ psi} \quad \text{and} \quad P_{\text{low}} = (10.7 \text{ kPa}) \left( \frac{1 \text{ psi}}{6.895 \text{kPa}} \right) = 1.55 \text{ psi}
\]

For a given pressure, the relation \( P = \rho gh \) is expressed for mercury and water as \( P = \rho_{\text{water}} gh_{\text{water}} \) and \( P = \rho_{\text{mercury}} gh_{\text{mercury}} \). Setting these two relations equal to each other and solving for water height gives

\[
P = \rho_{\text{water}} gh_{\text{water}} = \rho_{\text{mercury}} gh_{\text{mercury}} \quad \Rightarrow \quad h_{\text{water}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} h_{\text{mercury}}
\]

Therefore,

\[
h_{\text{water, high}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} h_{\text{mercury, high}} = \frac{13,600 \text{ kg/m}^3}{1000 \text{ kg/m}^3} (0.12 \text{ m}) = 1.63 \text{ m}
\]

\[
h_{\text{water, low}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} h_{\text{mercury, low}} = \frac{13,600 \text{ kg/m}^3}{1000 \text{ kg/m}^3} (0.08 \text{ m}) = 1.09 \text{ m}
\]

**Discussion** Note that measuring blood pressure with a water manometer would involve water column heights higher than the person’s height, and thus it is impractical. This problem shows why mercury is a suitable fluid for blood pressure measurement devices.

---

3-34

**Solution** A vertical tube open to the atmosphere is connected to the vein in the arm of a person. The height that the blood rises in the tube is to be determined.

**Assumptions** 1 The density of blood is constant. 2 The gage pressure of blood is 120 mmHg.

**Properties** The density of blood is given to be \( \rho = 1040 \text{ kg/m}^3 \).

**Analysis** For a given gage pressure, the relation \( P = \rho gh \) can be expressed for mercury and blood as \( P = \rho_{\text{blood}} gh_{\text{blood}} \) and \( P = \rho_{\text{mercury}} gh_{\text{mercury}} \). Setting these two relations equal to each other we get

\[
P = \rho_{\text{blood}} gh_{\text{blood}} = \rho_{\text{mercury}} gh_{\text{mercury}}
\]

Solving for blood height and substituting gives

\[
h_{\text{blood}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{blood}}} h_{\text{mercury}} = \frac{13,600 \text{ kg/m}^3}{1040 \text{ kg/m}^3} (0.12 \text{ m}) = 1.57 \text{ m}
\]

**Discussion** Note that the blood can rise about one and a half meters in a tube connected to the vein. This explains why IV tubes must be placed high to force a fluid into the vein of a patient.
3-35

**Solution**
A man is standing in water vertically while being completely submerged. The difference between the pressure acting on his head and the pressure acting on his toes is to be determined.

**Assumptions**
Water is an incompressible substance, and thus the density does not change with depth.

**Properties**
We take the density of water to be \( \rho = 1000 \text{ kg/m}^3 \).

**Analysis**
The pressures at the head and toes of the person can be expressed as

\[
P_{\text{head}} = P_{\text{atm}} + \rho \cdot gh_{\text{head}} \quad \text{and} \quad P_{\text{toe}} = P_{\text{atm}} + \rho \cdot gh_{\text{toe}}
\]

where \( h \) is the vertical distance of the location in water from the free surface. The pressure difference between the toes and the head is determined by subtracting the first relation above from the second,

\[
P_{\text{toe}} - P_{\text{head}} = \rho \cdot gh_{\text{toe}} - \rho \cdot gh_{\text{head}} = \rho g (h_{\text{toe}} - h_{\text{head}})
\]

Substituting,

\[
P_{\text{toe}} - P_{\text{head}} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.80 \text{ m} - 0) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{kPa}}{1000 \text{N/m}^2} \right) = 17.7 \text{ kPa}
\]

**Discussion**
This problem can also be solved by noting that the atmospheric pressure (1 atm = 101.325 kPa) is equivalent to 10.3-m of water height, and finding the pressure that corresponds to a water height of 1.8 m.

3-36

**Solution**
Water is poured into the U-tube from one arm and oil from the other arm. The water column height in one arm and the ratio of the heights of the two fluids in the other arm are given. The height of each fluid in that arm is to be determined.

**Assumptions**
Both water and oil are incompressible substances.

**Properties**
The density of oil is given to be \( \rho_{\text{oil}} = 790 \text{ kg/m}^3 \). We take the density of water to be \( \rho_w = 1000 \text{ kg/m}^3 \).

**Analysis**
The height of water column in the left arm of the manometer is given to be \( h_{w1} = 0.70 \text{ m} \). We let the height of water and oil in the right arm to be \( h_{w2} \) and \( h_a \), respectively. Then, \( h_a = 6h_{w2} \). Noting that both arms are open to the atmosphere, the pressure at the bottom of the U-tube can be expressed as

\[
P_{\text{bottom}} = P_{\text{atm}} + \rho_w \cdot gh_{w1} \quad \text{and} \quad P_{\text{bottom}} = P_{\text{atm}} + \rho_w \cdot gh_{w2} + \rho_o \cdot gh_a
\]

Setting them equal to each other and simplifying,

\[
\rho_w \cdot gh_{w1} = \rho_w \cdot gh_{w2} + \rho_o \cdot gh_a \quad \rightarrow \quad \rho_w \cdot h_{w1} = \rho_w \cdot h_{w2} + \rho_o \cdot h_a \quad \rightarrow \quad h_{w1} = h_{w2} + \rho_o / \rho_w \cdot h_a
\]

Noting that \( h_a = 6h_{w2} \) and we take \( \rho_o = \rho_{\text{oil}} \), the water and oil column heights in the second arm are determined to be

\[
0.7 \text{ m} = h_{w2} + (790/1000)6h_{w2} \quad \rightarrow \quad h_{w2} = 0.122 \text{ m}
\]

\[
0.7 \text{ m} = 0.122 \text{ m} + (790/1000)h_a \quad \rightarrow \quad h_a = 0.732 \text{ m}
\]

**Discussion**
Note that the fluid height in the arm that contains oil is higher. This is expected since oil is lighter than water.
3-37

**Solution**  The hydraulic lift in a car repair shop is to lift cars. The fluid gage pressure that must be maintained in the reservoir is to be determined.

**Assumptions**  The weight of the piston of the lift is negligible.

**Analysis**  Pressure is force per unit area, and thus the gage pressure required is simply the ratio of the weight of the car to the area of the lift,

\[
P_{\text{gage}} = \frac{W}{A} = \frac{mg}{\pi D^2 / 4} = \frac{(1800 \text{ kg})(9.81 \text{ m/s}^2)}{\pi(0.40 \text{ m})^2 / 4} \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 141 \text{ kN/m}^2 = 141 \text{kPa}
\]

**Discussion**  Note that the pressure level in the reservoir can be reduced by using a piston with a larger area.

3-38

**Solution**  Fresh and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer. The pressure difference between the two pipelines is to be determined.

**Assumptions**  1 All the liquids are incompressible.  2 The effect of air column on pressure is negligible.

**Properties**  The densities of seawater and mercury are given to be \( \rho_{\text{sea}} = 1035 \text{ kg/m}^3 \) and \( \rho_{\text{Hg}} = 13,600 \text{ kg/m}^3 \). We take the density of water to be \( \rho_w = 1000 \text{ kg/m}^3 \).

**Analysis**  Starting with the pressure in the fresh water pipe (point 1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the \( \rho gh \) terms until we reach the seawater pipe (point 2), and setting the result equal to \( P_2 \) gives

\[
P_1 + \rho_w g h_w - \rho_{\text{Hg}} g h_{\text{Hg}} - \rho_{\text{air}} g h_{\text{air}} + \rho_{\text{sea}} g h_{\text{sea}} = P_2
\]

Rearranging and neglecting the effect of air column on pressure,

\[
P_1 - P_2 = -\rho_w g h_w + \rho_{\text{Hg}} g h_{\text{Hg}} - \rho_{\text{sea}} g h_{\text{sea}} = g(\rho_{\text{Hg}} h_{\text{Hg}} - \rho_w h_w - \rho_{\text{sea}} h_{\text{sea}})
\]

Substituting,

\[
P_1 - P_2 = (9.81 \text{ m/s}^2)(13600 \text{ kg/m}^3)(0.1 \text{ m})
\]

\[
- (1000 \text{ kg/m}^3)(0.6 \text{ m}) - (1035 \text{ kg/m}^3)(0.4 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right)
\]

\[
= 3.39 \text{ kN/m}^2 = 3.39 \text{kPa}
\]

Therefore, the pressure in the fresh water pipe is 3.39 kPa higher than the pressure in the seawater pipe.

**Discussion**  A 0.70-m high air column with a density of 1.2 kg/m\(^3\) corresponds to a pressure difference of 0.008 kPa. Therefore, its effect on the pressure difference between the two pipes is negligible.
3-39

**Solution**  Fresh and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer. The pressure difference between the two pipelines is to be determined.

**Assumptions**  All the liquids are incompressible.

**Properties**  The densities of seawater and mercury are given to be $\rho_{\text{sea}} = 1035 \text{ kg/m}^3$ and $\rho_{\text{Hg}} = 13,600 \text{ kg/m}^3$. We take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$. The specific gravity of oil is given to be 0.72, and thus its density is $720 \text{ kg/m}^3$.

**Analysis**  Starting with the pressure in the fresh water pipe (point 1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the $\rho gh$ terms until we reach the sea water pipe (point 2), and setting the result equal to $P_2$ gives

$$P_1 + \rho_w gh_w - \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{oil}} gh_{\text{oil}} + \rho_{\text{sea}} gh_{\text{sea}} = P_2$$

Rearranging,

$$P_1 - P_2 = -\rho_w gh_w + \rho_{\text{Hg}} gh_{\text{Hg}} + \rho_{\text{oil}} gh_{\text{oil}} - \rho_{\text{sea}} gh_{\text{sea}} = g(\rho_{\text{Hg}} h_{\text{Hg}} + \rho_{\text{oil}} h_{\text{oil}} - \rho_w h_w - \rho_{\text{sea}} h_{\text{sea}})$$

Substituting,

$$P_1 - P_2 = (9.81 \text{ m/s}^2)[(13600 \text{ kg/m}^3)(0.1 \text{ m}) + (720 \text{ kg/m}^3)(0.7 \text{ m}) - (1000 \text{ kg/m}^3)(0.6 \text{ m}) - (1035 \text{ kg/m}^3)(0.4 \text{ m})] \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right)$$

$$= 8.34 \text{ kN/m}^2 = 8.34 \text{ kPa}$$

Therefore, the pressure in the fresh water pipe is 8.34 kPa higher than the pressure in the sea water pipe.

**Discussion**  The result is greater than that of the previous problem since the oil is heavier than the air.
3-40E  

**Solution**  
The pressure in a natural gas pipeline is measured by a double U-tube manometer with one of the arms open to the atmosphere. The absolute pressure in the pipeline is to be determined.

**Assumptions**  
1. All the liquids are incompressible.  
2. The effect of air column on pressure is negligible.  
3. The pressure throughout the natural gas (including the tube) is uniform since its density is low.

**Properties**  
We take the density of water to be \( \rho_w = 62.4 \text{ lbm/ft}^3 \). The specific gravity of mercury is given to be 13.6, and thus its density is \( \rho_{\text{Hg}} = 13.6 \times 62.4 = 848.6 \text{ lbm/ft}^3 \).

**Analysis**  
Starting with the pressure at point 1 in the natural gas pipeline, and moving along the tube by adding (as we go down) or subtracting (as we go up) the \( \rho g h \) terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to \( P_{\text{atm}} \) gives

\[
P_1 - \rho_{\text{Hg}} g h_{\text{Hg}} - \rho_w g h_{\text{water}} = P_{\text{atm}}
\]

Solving for \( P_1 \),

\[
P_1 = P_{\text{atm}} + \rho_{\text{Hg}} g h_{\text{Hg}} + \rho_w g h_{\text{water}}
\]

Substituting,

\[
P = 14.2 \text{ psia} + (32.2 \text{ ft/s}^2)(848.6 \text{ lbm/ft}^3)(6/12 \text{ ft}) + (62.4 \text{ lbm/ft}^3)(24/12 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right)
\]

\[
= 18.0 \text{ psia}
\]

**Discussion**  
Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly. Also, it can be shown that the 15-in high air column with a density of 0.075 lbm/ft\(^3\) corresponds to a pressure difference of 0.00065 psi. Therefore, its effect on the pressure difference between the two pipes is negligible.
3-41E

**Solution** The pressure in a natural gas pipeline is measured by a double U-tube manometer with one of the arms open to the atmosphere. The absolute pressure in the pipeline is to be determined.

**Assumptions**
1. All the liquids are incompressible.
2. The pressure throughout the natural gas (including the tube) is uniform since its density is low.

**Properties**
- We take the density of water to be \( \rho_w = 62.4 \text{ lbm/ft}^3 \). The specific gravity of mercury is given to be 13.6, and thus its density is \( \rho_{\text{Hg}} = 13.6 \times 62.4 = 848.6 \text{ lbm/ft}^3 \). The specific gravity of oil is given to be 0.69, and thus its density is \( \rho_{\text{oil}} = 0.69 \times 62.4 = 43.1 \text{ lbm/ft}^3 \).

**Analysis**

Starting with the pressure at point 1 in the natural gas pipeline, and moving along the tube by adding (as we go down) or subtracting (as we go up) the \( \rho g h \) terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to \( P_{\text{atm}} \) gives

\[
P_1 - \rho_{\text{Hg}} gh_{\text{Hg}} + \rho_{\text{oil}} gh_{\text{oil}} - \rho_{\text{water}} gh_{\text{water}} = P_{\text{atm}}
\]

Solving for \( P_1 \),

\[
P_1 = P_{\text{atm}} + \rho_{\text{Hg}} gh_{\text{Hg}} + \rho_{\text{water}} gh_1 - \rho_{\text{oil}} gh_{\text{oil}}
\]

Substituting,

\[
P_1 = 14.2 \text{ psia} + (32.2 \text{ ft/s}^2) [(848.6 \text{ lbm/ft}^3)(6/12 \text{ ft}) + (62.4 \text{ lbm/ft}^3)(27/12 \text{ ft})]
\]

\[
- (43.1 \text{ lbm/ft}^3)(15/12 \text{ ft}) \left( \frac{1\text{lbf}}{32.2\text{lbm} \cdot \text{ft/s}^2} \right) \left( \frac{1\text{ft}^2}{144\text{in}^2} \right)
\]

\[
= 17.7 \text{ psia}
\]

**Discussion** Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly.
Solution The gage pressure of air in a pressurized water tank is measured simultaneously by both a pressure gage and a manometer. The differential height $h$ of the mercury column is to be determined.

Assumptions The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus the pressure at the air-water interface is the same as the indicated gage pressure.

Properties We take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$. The specific gravities of oil and mercury are given to be 0.72 and 13.6, respectively.

Analysis Starting with the pressure of air in the tank (point 1), and moving along the tube by adding (as we go down) or subtracting (as we go up) the $\rho gh$ terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to $P_{atm}$ gives

$$P_1 + \rho_w gh_w - \rho_{Hg} gh_{Hg} - \rho_{oil} gh_{oil} = P_{atm}$$

Rearranging,

$$P_1 - P_{atm} = \rho_{oil} gh_{oil} + \rho_{Hg} gh_{Hg} - \rho_w gh_w$$

or,

$$\frac{P_{gage}}{\rho_w g} = \rho_{s,oil} h_{oil} + \rho_{s,Hg} h_{Hg} - h_w$$

Substituting,

$$\left( \frac{65 \text{ kPa}}{1000 \text{ kg/m}^3} \right) \left( \frac{1000 \text{ kg/m}^3 \cdot m/s^2}{1 \text{ kPa} \cdot m^2} \right) = 0.72 \times (0.75 \text{ m}) + 13.6 \times h_{Hg} - 0.3 \text{ m}$$

Solving for $h_{Hg}$ gives $h_{Hg} = 0.47 \text{ m}$. Therefore, the differential height of the mercury column must be 47 cm.

Discussion Double instrumentation like this allows one to verify the measurement of one of the instruments by the measurement of another instrument.
3-43

Solution  The gage pressure of air in a pressurized water tank is measured simultaneously by both a pressure gage and a manometer. The differential height $h$ of the mercury column is to be determined.

Assumptions  The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus the pressure at the air-water interface is the same as the indicated gage pressure.

Properties  We take the density of water to be $\rho_w=1000 \text{ kg/m}^3$. The specific gravities of oil and mercury are given to be 0.72 and 13.6, respectively.

Analysis  Starting with the pressure of air in the tank (point 1), and moving along the tube by adding (as we go down) or subtracting (as we go up) the $\rho gh$ terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to $P_{atm}$ gives

$$P_1 + \rho_w gh_w - \rho_{Hg} gh_{Hg} - \rho_{oil} gh_{oil} = P_{atm}$$

Rearranging,

$$P_1 - P_{atm} = \rho_{oil} gh_{oil} + \rho_{Hg} gh_{Hg} - \rho_w gh_w$$

or,

$$\frac{P_{gage}}{\rho_w g} = SG_{oil} h_{oil} + SG_{Hg} h_{Hg} - h_w$$

Substituting,

$$\frac{45 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\left(\frac{1000 \text{ kg}\cdot\text{m/s}^2}{1 \text{kPa}\cdot\text{m}^2}\right)} = 0.72 \times (0.75 \text{ m}) + 13.6 \times h_{Hg} - 0.3 \text{ m}$$

Solving for $h_{Hg}$ gives $h_{Hg} = 0.32 \text{ m}$. Therefore, the differential height of the mercury column must be 32 cm.

Discussion  Double instrumentation like this allows one to verify the measurement of one of the instruments by the measurement of another instrument.

3-44

Solution  The top part of a water tank is divided into two compartments, and a fluid with an unknown density is poured into one side. The levels of the water and the liquid are measured. The density of the fluid is to be determined.

Assumptions  1 Both water and the added liquid are incompressible substances. 2 The added liquid does not mix with water.

Properties  We take the density of water to be $\rho =1000 \text{ kg/m}^3$.

Analysis  Both fluids are open to the atmosphere. Noting that the pressure of both water and the added fluid is the same at the contact surface, the pressure at this surface can be expressed as

$$P_{contact} = P_{atm} + \rho_f gh_f = P_{atm} + \rho_w gh_w$$

Simplifying, we have $\rho_f gh_f = \rho_w gh_w$. Solving for $\rho_f$ gives

$$\rho_f = \frac{h_w}{h_f} \rho_w = \frac{50 \text{ cm}}{75 \text{ cm}} \left(\frac{1000 \text{ kg/m}^3}{1000 \text{ kg/m}^3}\right) = 667 \text{ kg/m}^3$$

Discussion  Note that the added fluid is lighter than water as expected (a heavier fluid would sink in water).
Solution

A load on a hydraulic lift is to be raised by pouring oil from a thin tube. The height of oil in the tube required in order to raise that weight is to be determined.

Assumptions
1. The cylinders of the lift are vertical.
2. There are no leaks.
3. Atmospheric pressure acts on both sides, and thus it can be disregarded.

Properties
The density of oil is given to be \( \rho = 780 \text{ kg/m}^3 \).

Analysis
Noting that pressure is force per unit area, the gage pressure in the fluid under the load is simply the ratio of the weight to the area of the lift,

\[
\begin{align*}
P_{\text{gage}} &= \frac{W}{A} = \frac{mg}{\pi D^2 / 4} = \frac{(500 \text{ kg})(9.81 \text{ m/s}^2)}{\pi (1.20 \text{ m})^2 / 4} \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 4.34 \text{ kN/m}^2 = 4.34 \text{ kPa}
\end{align*}
\]

The required oil height that will cause 4.34 kPa of pressure rise is

\[
\begin{align*}
P_{\text{gage}} &= \rho gh \rightarrow \ h = \frac{P_{\text{gage}}}{\rho g} = \frac{4.34 \text{ kN/m}^2}{(780 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kN/m}^2} \right) = 0.567 \text{ m}
\end{align*}
\]

Therefore, a 500 kg load can be raised by this hydraulic lift by simply raising the oil level in the tube by 56.7 cm.

Discussion
Note that large weights can be raised by little effort in hydraulic lift by making use of Pascal’s principle.
Solution  Two oil tanks are connected to each other through a mercury manometer. For a given differential height, the pressure difference between the two tanks is to be determined.

Assumptions  1 Both the oil and mercury are incompressible fluids. 2 The oils in both tanks have the same density.

Properties  The densities of oil and mercury are given to be \( \rho_{\text{oil}} = 45 \text{ lbm/ft}^3 \) and \( \rho_{\text{Hg}} = 848 \text{ lbm/ft}^3 \).

Analysis  Starting with the pressure at the bottom of tank 1 (where pressure is \( P_1 \)) and moving along the tube by adding (as we go down) or subtracting (as we go up) the \( \rho gh \) terms until we reach the bottom of tank 2 (where pressure is \( P_2 \)) gives

\[
P_1 + \rho_{\text{oil}} g (h_1 + h_2) - \rho_{\text{Hg}} g h_2 - \rho_{\text{oil}} g h_1 = P_2
\]

where \( h_1 = 10 \text{ in} \) and \( h_2 = 32 \text{ in} \). Rearranging and simplifying,

\[
P_1 - P_2 = \rho_{\text{Hg}} g h_2 - \rho_{\text{oil}} g h_1 = (\rho_{\text{Hg}} - \rho_{\text{oil}}) g h_2
\]

Substituting,

\[
\Delta P = P_1 - P_2 = (848 - 45 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(32/12 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm-ft/s}^2} \right) \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) = 14.9 \text{ psia}
\]

Therefore, the pressure in the left oil tank is 14.9 psia higher than the pressure in the right oil tank.

Discussion  Note that large pressure differences can be measured conveniently by mercury manometers. If a water manometer were used in this case, the differential height would be over 30 ft.

3-47

Solution  The standard atmospheric pressure is expressed in terms of mercury, water, and glycerin columns.

Assumptions  The densities of fluids are constant.

Properties  The specific gravities are given to be \( \text{SG} = 13.6 \) for mercury, \( \text{SG} = 1.0 \) for water, and \( \text{SG} = 1.26 \) for glycerin. The standard density of water is 1000 kg/m³, and the standard atmospheric pressure is 101,325 Pa.

Analysis  The atmospheric pressure is expressed in terms of a fluid column height as

\[
P_{\text{atm}} = \rho g h = \text{SG} \rho \text{w} g h \rightarrow h = \frac{P_{\text{atm}}}{\text{SG} \rho \text{w} g}
\]

Substituting,

(a) Mercury:  \( h = \frac{P_{\text{atm}}}{\text{SG} \rho \text{w} g} = \frac{101,325 \text{ N/m}^2}{13.6(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kg-m/s}^2}{1 \text{ N/m}^2} \right) = 0.759 \text{ m} \)

(b) Water:  \( h = \frac{P_{\text{atm}}}{\text{SG} \rho \text{w} g} = \frac{101,325 \text{ N/m}^2}{1(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kg-m/s}^2}{1 \text{ N/m}^2} \right) = 10.3 \text{ m} \)

(c) Glycerin:  \( h = \frac{P_{\text{atm}}}{\text{SG} \rho \text{w} g} = \frac{101,325 \text{ N/m}^2}{1.26(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kg-m/s}^2}{1 \text{ N/m}^2} \right) = 8.20 \text{ m} \)

Discussion  Using water or glycerin to measure atmospheric pressure requires very long vertical tubes (over 10 m for water), which is not practical. This explains why mercury is used instead of water or a light fluid.
Solution A glass filled with water and covered with a thin paper is inverted. The pressure at the bottom of the glass is to be determined.

Assumptions 1 Water is an incompressible substance. 2 The weight of the paper is negligible. 3 The atmospheric pressure is 100 kPa.

Properties We take the density of water to be $\rho = 1000 \text{ kg/m}^3$.

Analysis The paper is in equilibrium, and thus the net force acting on the paper must be zero. A vertical force balance on the paper involves the pressure forces on both sides, and yields

$$P_1 A_{\text{glass}} = P_{\text{atm}} A_{\text{glass}} \quad \Rightarrow \quad P_1 = P_{\text{atm}}$$

That is, the pressures on both sides of the paper must be the same.

The pressure at the bottom of the glass is determined from the hydrostatic pressure relation to be

$$P_{\text{atm}} = P_{\text{bottom}} + \rho gh_{\text{glass}} \quad \Rightarrow \quad P_{\text{bottom}} = P_{\text{atm}} - \rho gh_{\text{glass}}$$

Substituting,

$$P_{\text{bottom}} = (100 \text{ kPa}) - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.12 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) = 98.8 \text{ kPa}$$

Discussion Note that there is a vacuum of 1 kPa at the bottom of the glass, and thus there is an upward pressure force acting on the water body, which balanced by the weight of water. As a result, the net downward force on water is zero, and thus water does not flow down.
Solution Two chambers with the same fluid at their base are separated by a piston. The gage pressure in each air chamber is to be determined.

Assumptions 1 Water is an incompressible substance. 2 The variation of pressure with elevation in each air chamber is negligible because of the low density of air.

Properties We take the density of water to be $\rho =1000$ kg/m$^3$.

Analysis The piston is in equilibrium, and thus the net force acting on the piston must be zero. A vertical force balance on the piston involves the pressure force exerted by water on the piston face, the atmospheric pressure force, and the piston weight, and yields

$$P_c A_{\text{piston}} = P_{\text{atm}} A_{\text{piston}} + W_{\text{piston}} \quad \rightarrow \quad P_c = P_{\text{atm}} + \frac{W_{\text{piston}}}{A_{\text{piston}}}$$

The pressure at the bottom of each air chamber is determined from the hydrostatic pressure relation to be

$$P_{\text{air } A} = P_E = P_C + \rho g CE = P_{\text{atm}} + \frac{W_{\text{piston}}}{A_{\text{piston}}} + \rho g CE \quad \rightarrow \quad P_{\text{air } A, \text{gage}} = \frac{W_{\text{piston}}}{A_{\text{piston}}} + \rho g CE$$

$$P_{\text{air } B} = P_D = P_C - \rho g CD = P_{\text{atm}} + \frac{W_{\text{piston}}}{A_{\text{piston}}} - \rho g CD \quad \rightarrow \quad P_{\text{air } B, \text{gage}} = \frac{W_{\text{piston}}}{A_{\text{piston}}} - \rho g CD$$

Substituting,

$$P_{\text{air } A, \text{gage}} = \frac{25 \text{ N}}{\pi (0.3 \text{ m})^2 / 4} + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.25 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 2806 \text{ N/m}^2 = 2.81 \text{ kPa}$$

$$P_{\text{air } B, \text{gage}} = \frac{25 \text{ N}}{\pi (0.3 \text{ m})^2 / 4} - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.25 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = -2099 \text{ N/m}^2 = -2.10 \text{ kPa}$$

Discussion Note that there is a vacuum of about 2 kPa in tank B which pulls the water up.
Chapter 3 Pressure and Fluid Statics

3-50

Solution  A double-fluid manometer attached to an air pipe is considered. The specific gravity of one fluid is known, and the specific gravity of the other fluid is to be determined.

Assumptions  1 Densities of liquids are constant. 2 The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus the pressure at the air-water interface is the same as the indicated gage pressure.

Properties  The specific gravity of one fluid is given to be 13.55. We take the standard density of water to be 1000 kg/m³.

Analysis  Starting with the pressure of air in the tank, and moving along the tube by adding (as we go down) or subtracting (as we go up) the \( \rho gh \) terms until we reach the free surface where the oil tube is exposed to the atmosphere, and setting the result equal to \( P_{\text{atm}} \) give

\[
P_{\text{air}} + \rho_1 gh_1 - \rho_2 gh_2 = P_{\text{atm}} \quad \rightarrow \quad P_{\text{air}} - P_{\text{atm}} = SG_2 \rho_a gh_2 - SG_1 \rho_a gh_1
\]

Rearranging and solving for \( SG_2 \),

\[
SG_2 = SG_1 \frac{h_1}{h_2} + \frac{P_{\text{air}} - P_{\text{atm}}}{\rho_a gh_2} = 13.55 \times \frac{0.22 \text{ m}}{0.40 \text{ m}} + \left( \frac{76 - 100}{1000 \text{ kg/m}^3}(9.81 \text{ m/s}^2)(0.40 \text{ m}) \right) \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kPa} \cdot \text{m}} \right) = 1.3363 \approx 1.34
\]

Discussion  Note that the right fluid column is higher than the left, and this would imply above atmospheric pressure in the pipe for a single-fluid manometer.
Chapter 3 Pressure and Fluid Statics

Solution  The pressure difference between two pipes is measured by a double-fluid manometer. For given fluid heights and specific gravities, the pressure difference between the pipes is to be calculated.

Assumptions  All the liquids are incompressible.

Properties  The specific gravities are given to be 13.5 for mercury, 1.26 for glycerin, and 0.88 for oil. We take the standard density of water to be $\rho_w = 1000 \text{ kg/m}^3$.

Analysis  Starting with the pressure in the water pipe (point A) and moving along the tube by adding (as we go down) or subtracting (as we go up) the $\rho gh$ terms until we reach the oil pipe (point B), and setting the result equal to $P_B$ give

$$P_A + \rho_w gh_w + \rho_{1\text{Hg}} gh_{1\text{Hg}} - \rho_{\text{gly}} gh_{\text{gly}} + \rho_{\text{oil}} gh_{\text{oil}} = P_B$$

Rearranging and using the definition of specific gravity,

$$P_B - P_A = \rho_w gh_w + \rho_{1\text{Hg}} gh_{1\text{Hg}} - \rho_{\text{gly}} gh_{\text{gly}} + \rho_{\text{oil}} gh_{\text{oil}}$$

Substituting,

$$P_B - P_A = (9.81 \text{ m/s}^2)(1000 \text{ kg/m}^3)(1(0.55 \text{ m}) + 13.5(0.2 \text{ m}) - 1.26(0.42 \text{ m}) + 0.88(0.1 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right)$$

$$= 27.6 \text{ kN/m}^2 = 27.6 \text{ kPa}$$

Therefore, the pressure in the oil pipe is 27.6 kPa higher than the pressure in the water pipe.

Discussion  Using a manometer between two pipes is not recommended unless the pressures in the two pipes are relatively constant. Otherwise, an over-rise of pressure in one pipe can push the manometer fluid into the other pipe, creating a short circuit.
Solution The fluid levels in a multi-fluid U-tube manometer change as a result of a pressure drop in the trapped air space. For a given pressure drop and brine level change, the area ratio is to be determined.

Assumptions 1 All the liquids are incompressible. 2 Pressure in the brine pipe remains constant. 3 The variation of pressure in the trapped air space is negligible.

Properties The specific gravities are given to be 13.56 for mercury and 1.1 for brine. We take the standard density of water to be $\rho_{w} = 1000 \text{ kg/m}^3$.

Analysis It is clear from the problem statement and the figure that the brine pressure is much higher than the air pressure, and when the air pressure drops by 0.7 kPa, the pressure difference between the brine and the air space also increases by the same amount. Starting with the air pressure (point A) and moving along the tube by adding (as we go down) or subtracting (as we go up) the $\rho g h$ terms until we reach the brine pipe (point B), and setting the result equal to $P_B$ before and after the pressure change of air give

Before:  
$$P_{A1} + \rho_w g h_w + \rho_{Hg} g h_{Hg,1} - \rho_{br} g h_{br,1} = P_B$$

After:  
$$P_{A2} + \rho_w g h_w + \rho_{Hg} g h_{Hg,2} - \rho_{br} g h_{br,2} = P_B$$

Subtracting,

$$P_{A2} - P_{A1} + \rho_{Hg} g \Delta h_{Hg} - \rho_{br} g \Delta h_{br} = 0 \rightarrow \frac{P_{A2} - P_{A1}}{\rho_{Hg} g} = \frac{\rho_{Hg} \Delta h_{Hg} - \rho_{br} \Delta h_{br}}{\rho_{br} g} = 0 \quad (1)$$

where $\Delta h_{Hg}$ and $\Delta h_{br}$ are the changes in the differential mercury and brine column heights, respectively, due to the drop in air pressure. Both of these are positive quantities since as the mercury-brine interface drops, the differential fluid heights for both mercury and brine increase. Noting also that the volume of mercury is constant, we have $A_1 \Delta h_{Hg, left} = A_2 \Delta h_{Hg, right}$ and

$$P_{A2} - P_{A1} = -0.7 \text{ kPa} = -700 \text{ N/m}^2 = -700 \text{ kg/m}^2 \text{ s}^2$$

$$\Delta h_{br} = 0.005 \text{ m}$$

$$\Delta h_{Hg} = \Delta h_{Hg, right} + \Delta h_{Hg, left} = \Delta h_{br} + \Delta h_{br} A_2/A_1 = \Delta h_{br} (1 + A_2/A_1)$$

Substituting,

$$\frac{700 \text{ kg/m}^2 \text{ s}^2}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = \frac{13.56 \times 0.005(1 + A_2/A_1) - 1.1 \times 0.005}{\text{ m}}$$

It gives

$$A_2/A_1 = 0.134$$

Discussion In addition to the equations of hydrostatics, we also utilize conservation of mass in this problem.
Two water tanks are connected to each other through a mercury manometer with inclined tubes. For a given pressure difference between the two tanks, the parameters $a$ and $\theta$ are to be determined.

**Assumptions**  Both water and mercury are incompressible liquids.

**Properties**  The specific gravity of mercury is given to be 13.6. We take the standard density of water to be $\rho_w = 1000$ kg/m$^3$.

**Analysis**  Starting with the pressure in the tank $A$ and moving along the tube by adding (as we go down) or subtracting (as we go up) the $\rho g h$ terms until we reach tank $B$, and setting the result equal to $P_B$, give

$$P_A + \rho_w ga + \rho_{Hg} g 2a - \rho_w ga = P_B$$

$$2 \rho_{Hg} ga = P_B - P_A$$

Rearranging and substituting the known values,

$$a = \frac{P_B - P_A}{2 \rho_{Hg} g} = \frac{20 \text{kN/m}^2}{2(13.6)(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{kN}} \right) = 0.0750 \text{ m} = 7.50 \text{ cm}$$

From geometric considerations,

$$26.8 \sin \theta = 2a \quad \text{(cm)}$$

Therefore,

$$\sin \theta = \frac{2a}{26.8} = \frac{2 \times 7.50}{26.8} = 0.560 \quad \rightarrow \quad \theta = 34.0^\circ$$

**Discussion**  Note that vertical distances are used in manometer analysis. Horizontal distances are of no consequence.
Solution  A multi-fluid container is connected to a U-tube. For the given specific gravities and fluid column heights, the gage pressure at A and the height of a mercury column that would create the same pressure at A are to be determined.

Assumptions  1 All the liquids are incompressible. 2 The multi-fluid container is open to the atmosphere.

Properties  The specific gravities are given to be 1.26 for glycerin and 0.90 for oil. We take the standard density of water to be \( \rho_w = 1000 \text{ kg/m}^3 \), and the specific gravity of mercury to be 13.6.

Analysis  Starting with the atmospheric pressure on the top surface of the container and moving along the tube by adding (as we go down) or subtracting (as we go up) the \( \rho gh \) terms until we reach point A, and setting the result equal to \( P_A \) give

\[
P_{atm} + \rho_{oil}gh_{oil} + \rho_{w}gh_{w} - \rho_{gly}gh_{gly} = P_A
\]

Rearranging and using the definition of specific gravity,

\[
P_A - P_{atm} = SG_{oil}\rho_{oil}gh_{oil} + SG_{w}\rho_{w}gh_{w} - SG_{gly}\rho_{gly}gh_{gly}
\]

or

\[
P_{A,gage} = \rho_{w}\left(SG_{oil}h_{oil} + SG_{w}h_{w} - SG_{gly}h_{gly}\right)
\]

Substituting,

\[
P_{A,gage} = (9.81 \text{ m/s}^2)(1000 \text{ kg/m}^3)[0.90(0.80 \text{ m}) + 1(0.35 \text{ m}) - 1.26(0.72 \text{ m})]\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)
\]

\[
= 1.5971 \text{ kN/m}^2 \cong 1.60 \text{ kPa}
\]

The equivalent mercury column height is

\[
h_{Hg} = \frac{P_{A,gage}}{\rho_{Hg}g} = \frac{1.5971 \text{ kN/m}^2}{(13.6)(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kN}}\right) = 0.011971 \text{ m} \cong 1.20 \text{ cm}
\]

Discussion  Note that the high density of mercury makes it a very suitable fluid for measuring high pressures in manometers.
Solution  We are to determine the force required to lift a car with a hydraulic jack at two different elevations.

Assumptions  1 The oil is incompressible.  2 The system is at rest during the analysis (hydrostatics).

Analysis  (a) When \( h = 0 \), the pressure at the bottom of each piston must be the same. Thus,

\[
P_1 = \frac{F_1}{A_1} = P_2 = \frac{F_2}{A_2} \quad \rightarrow \quad F_1 = F_2 \frac{A_1}{A_2} = (20,000 \text{ N}) \frac{1.00 \text{ cm}^2}{0.0400 \text{ m}^2} \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)^2 = 50.0 \text{ N}
\]

At the beginning, when \( h = 0 \), the required force is thus \( F_1 = 50.0 \text{ N} \).

(b) When \( h \neq 0 \), the hydrostatic pressure due to the elevation difference must be taken into account, namely,

\[
P_1 = \frac{F_1}{A_1} = P_2 + \rho gh = \frac{F_2}{A_2} + \rho gh
\]

\[
F_1 = F_2 \frac{A_1}{A_2} + \rho gh A_1
\]

\[
= (20,000 \text{ N}) \frac{0.000100 \text{ m}^2}{0.0400 \text{ m}^2} + \left( 870 \frac{\text{kg}}{\text{m}^3} \right) \left( 9.807 \frac{\text{m}}{\text{s}^2} \right) (2.00 \text{ m}) (0.000100 \text{ m}^2) \left( \frac{\text{N}}{\text{kg} \cdot \text{m/s}^2} \right)
\]

\[
= 51.706 \text{ N}
\]

Thus, after the car has been raised 2 meters, the required force is \( F_1 = 51.7 \text{ N} \).

Comparing the two results, it takes more force to keep the car elevated than it does to hold it at \( h = 0 \). This makes sense physically because the elevation difference generates a higher pressure (and thus a higher required force) at the lower piston due to hydrostatics.

Discussion  When \( h = 0 \), the specific gravity (or density) of the hydraulic fluid does not enter the calculation – the problem simplifies to setting the two pressure equal. However, when \( h \neq 0 \), there is a hydrostatical head and therefore the density of the fluid enters the calculation.
Solution  We are to calculate the force produced by a hydraulic jack for various area ratios, and we are to compare our manual calculations to those generated using CFD.

Assumptions  1 The oil is incompressible. 2 The system is at rest during the analysis (hydrostatics).

Analysis  (a) When \( h = 0 \), the pressure at the bottom of each piston must be the same. Thus,

\[
P_1 = \frac{F_1}{A_1} = P_2 = \frac{F_2}{A_2} \quad \rightarrow \quad F_2 = F_1 \frac{A_1}{A_2}
\]

For example, when \( A_2/A_1 = 400 \) and \( F_1 = 50.0 \text{ N} \), the output force \( F_2 \) is simply \( F_2 = F_1 A_2/A_1 = (50.0 \text{ N})(400) = 20,000 \text{ N} \). Since \( F_1 \) is constant here, we see that \( F_2 \) varies linearly with area ratio \( A_2/A_1 \).

(b) The FlowLab template was run with \( F_1 = 50.0 \text{ N} \) and \( 50 < A_2/A_1 < 1000 \). The results are compared with the manual calculations in the table and plot shown below.

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<thead>
<tr>
<th>( A_2/A_1 )</th>
<th>Manual calculation</th>
<th>FlowLab calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>2500</td>
<td>2500</td>
</tr>
<tr>
<td>100</td>
<td>5000</td>
<td>5000</td>
</tr>
<tr>
<td>400</td>
<td>20000</td>
<td>20000</td>
</tr>
<tr>
<td>1000</td>
<td>50000</td>
<td>50000</td>
</tr>
</tbody>
</table>

The agreement between the manual and CFD calculations is excellent as expected since this is a simple hydrostatics problem. We also verify on the plot that the relationship between \( F_2 \) and \( A_2/A_1 \) is indeed linear.

Discussion  The specific gravity of the hydraulic oil is irrelevant in this problem since \( h = 0 \) (there is no elevation difference between the two pistons). The larger the area ratio of the hydraulic jack, the easier it is to use. In other words, the same supplied force \( F_1 \) can produce a much greater force \( F_2 \) to jack up the car.
Solution  We are to calculate the force produced by a hydraulic jack for various hydraulic fluids (various densities), and we are to compare our manual calculations to those generated using CFD.

Assumptions  1 The oil is incompressible. 2 The system is at rest during the analysis (hydrostatics).

Analysis  (a) When \( h = 0 \), the pressure at the bottom of each piston must be the same. Thus,

\[ P_1 = \frac{F_1}{A_1} = P_2 = \frac{F_2}{A_2} \rightarrow F_2 = F_1 \frac{A_2}{A_1} \]

For example, when \( A_2/A_1 = 400 \) and \( F_1 = 50.0 \text{ N} \), the output force \( F_2 \) is simply \( F_2 = F_1 A_2/A_1 = (50.0 \text{ N})(400) = 20,000 \text{ N} \). Since \( F_1 \) and area ratio \( A_2/A_1 \) are constants here, we see that indeed, \( F_2 \) does not vary with density. Thus, for any liquid, \( F_2 = 20,000 \text{ N} \).

(b) The FlowLab template was run with three liquids: \( \rho = 780.0 \text{ kg/m}^3 \) (kerosene), \( \rho = 870.0 \text{ kg/m}^3 \) (hydraulic oil), and \( \rho = 998.2 \text{ kg/m}^3 \) (water). The results are consistent with the hand calculations, namely, \( F_2 = 20,000 \text{ N} \) regardless of the liquid density.

Discussion  The specific gravity of the hydraulic oil is irrelevant in this problem since \( h = 0 \) (there is no elevation difference between the two pistons). However, if \( h \neq 0 \), there would be a hydrostatic head and therefore the density of the fluid would enter the calculation.
Solution  We are to calculate the force produced by a hydraulic jack for various values of input force $F_1$, and we are to compare our manual calculations to those generated using CFD.

Assumptions  
1. The oil is incompressible.  
2. The system is at rest during the analysis (hydrostatics).

Analysis  
(a) When elevation difference $h = 0$, the pressure at the bottom of each piston must be the same. Thus,

$$P_1 = \frac{F_1}{A_1} = P_2 = \frac{F_2}{A_2} \quad \rightarrow \quad F_2 = F_1 \frac{A_2}{A_1}$$

For example, when $A_2/A_1 = 400$ and $F_1 = 50.0 \text{ N}$, the output force $F_2$ is calculated as

$$F_2 = F_1 \frac{A_2}{A_1} = (10.0 \text{ N}) \frac{0.0400 \text{ m}^2}{0.000100 \text{ m}^2} = 4000 \text{ N}$$

Since $F_1, A_2/A_1,$ and $h$ are constants here, we see that $F_2$ varies linearly with applied force $F_1$.

(b) The FlowLab template was run with $A_2/A_1 = 400$, $\rho = 870.0 \text{ kg/m}^3$, $h = 0 \text{ m}$, and $10 \text{ N} < F_1 < 500 \text{ N}$. The results are compared with the manual calculations in the table and plot shown below.

<table>
<thead>
<tr>
<th>$F_1$ (N)</th>
<th>Manual calculation</th>
<th>FlowLab calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$F_2 = F_1 A_2/A_1$</td>
<td>4000</td>
</tr>
<tr>
<td>50</td>
<td>20000</td>
<td>20000</td>
</tr>
<tr>
<td>100</td>
<td>40000</td>
<td>40000</td>
</tr>
<tr>
<td>200</td>
<td>80000</td>
<td>80000</td>
</tr>
<tr>
<td>500</td>
<td>200000</td>
<td>200000</td>
</tr>
</tbody>
</table>

The agreement between the manual and CFD calculations is excellent as expected since this is a simple hydrostatics problem. We also verify on the plot that the relationship between $F_2$ and $F_1$ is indeed linear.

Discussion  
The specific gravity of the hydraulic oil is relevant in this problem since $h \neq 0$ (there is an elevation difference between the two pistons, and therefore the hydrostatic pressure head decreases the output force compared to the case with $h = 0$). When $F_1$ is small, this hydrostatic effect is more significant than when $F_1$ is large.
Solution  We are to calculate the force produced by a hydraulic jack for various values of elevation difference (height) \( h \), and we are to compare our manual calculations to those generated using CFD.

Assumptions  1. The oil is incompressible. 2. The system is at rest during the analysis (hydrostatics).

Analysis  
\( (a) \) When \( h \neq 0 \), the hydrostatic pressure due to the elevation difference must be taken into account, namely,

\[
P_1 = \frac{F_1}{A_1} = P_2 + \rho gh = \frac{F_2}{A_2} + \rho gh \quad \rightarrow \quad F_2 = F_1 \frac{A_2}{A_1} - \rho gh A_2
\]

For example, when \( A_2/A_1 = 400 \), \( h = 2.00 \text{ m} \), and \( F_1 = 50.0 \text{ N} \), the output force \( F_2 \) is calculated as

\[
F_2 = F_1 \frac{A_2}{A_1} - \rho gh A_2
\]

\[
= (50.0 \text{ N}) \frac{0.0400 \text{ m}^2}{0.000100 \text{ m}^2} - \left(870.0 \frac{\text{kg}}{\text{m}^3}\right) \left(9.807 \frac{\text{m}}{\text{s}^2}\right) (2.00 \text{ m}) (0.0400 \text{ m}^2) \left(\frac{\text{N}}{\text{kg} \cdot \text{m/s}^2}\right)
\]

\[
= 19,317.4 \text{ N} \approx 19,300 \text{ N} \quad \text{(to 3 significant digits)}
\]

Since \( F_1, A_2/A_1, \) and \( A_2 \) are constants here, we see that \( F_2 \) varies linearly with height \( h \) (with a negative slope).

\( (b) \) The FlowLab template was run with \( A_2/A_1 = 400, \rho = 870.0 \text{ kg/m}^3, F_1 = 50.0 \text{ N}, \) and \( 0 < h < 4.00 \text{ m} \). The results are compared with the manual calculations in the table and plot shown below.

<table>
<thead>
<tr>
<th>( h \text{ (m)} )</th>
<th>Manual calculation ( F_2 = F_1 A_2/A_1 - \rho gh A_2 \text{ (N)} )</th>
<th>FlowLab calculation ( F_2 \text{ (N)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0000</td>
<td>20000</td>
</tr>
<tr>
<td>1</td>
<td>19658.7164</td>
<td>19585.6</td>
</tr>
<tr>
<td>2</td>
<td>19317.4328</td>
<td>19317.2</td>
</tr>
<tr>
<td>3</td>
<td>18976.1492</td>
<td>18975.8</td>
</tr>
<tr>
<td>4</td>
<td>18634.8656</td>
<td>18634.4</td>
</tr>
</tbody>
</table>

The agreement between the manual and CFD calculations is excellent as expected since this is a simple hydrostatics problem. We also verify on the plot that the relationship between \( F_2 \) and \( h \) is indeed linear (with a negative slope).

Discussion  The slope is negative because as \( h \) increases, there is a greater loss of pressure at location 2 (the larger piston) due to hydrostatic pressure change.
Fluid Statics: Hydrostatic Forces on Plane and Curved Surfaces

3-60C  Solution  We are to define resultant force and center of pressure.

Analysis  The resultant hydrostatic force acting on a submerged surface is the resultant of the pressure forces acting on the surface. The point of application of this resultant force is called the center of pressure.

Discussion  The center of pressure is generally not at the center of the body, due to hydrostatic pressure variation.

3-61C  Solution  We are to examine a claim about hydrostatic force.

Analysis  Yes, because the magnitude of the resultant force acting on a plane surface of a completely submerged body in a homogeneous fluid is equal to the product of the pressure $P_c$ at the centroid of the surface and the area $A$ of the surface. The pressure at the centroid of the surface is $P_c = P_0 + \rho g h_C$, where $h_C$ is the vertical distance of the centroid from the free surface of the liquid.

Discussion  We have assumed that we also know the pressure at the liquid surface.

3-62C  Solution  We are to consider the effect of plate rotation on the hydrostatic force on the plate surface.

Analysis  There will be no change on the hydrostatic force acting on the top surface of this submerged horizontal flat plate as a result of this rotation since the magnitude of the resultant force acting on a plane surface of a completely submerged body in a homogeneous fluid is equal to the product of the pressure $P_c$ at the centroid of the surface and the area $A$ of the surface.

Discussion  If the rotation were not around the centroid, there would be a change in the force.

3-63C  Solution  We are to explain why dams are bigger at the bottom than at the top.

Analysis  Dams are built much thicker at the bottom because the pressure force increases with depth, and the bottom part of dams are subjected to largest forces.

Discussion  Dam construction requires an enormous amount of concrete, so tapering the dam in this way saves a lot of concrete, and therefore a lot of money.

3-64C  Solution  We are to explain how to determine the horizontal component of hydrostatic force on a curved surface.

Analysis  The horizontal component of the hydrostatic force acting on a curved surface is equal (in both magnitude and the line of action) to the hydrostatic force acting on the vertical projection of the curved surface.

Discussion  We could also integrate pressure along the surface, but the method discussed here is much simpler and yields the same answer.
3-65C

**Solution**

We are to explain how to determine the vertical component of hydrostatic force on a curved surface.

**Analysis**

The vertical component of the hydrostatic force acting on a curved surface is equal to the hydrostatic force acting on the horizontal projection of the curved surface, plus (minus, if acting in the opposite direction) the weight of the fluid block.

**Discussion**

We could also integrate pressure along the surface, but the method discussed here is much simpler and yields the same answer.

---

3-66C

**Solution**

We are to explain how to determine the line of action on a circular surface.

**Analysis**

The resultant hydrostatic force acting on a circular surface always passes through the center of the circle since the pressure forces are normal to the surface, and all lines normal to the surface of a circle pass through the center of the circle. Thus the pressure forces form a concurrent force system at the center, which can be reduced to a single equivalent force at that point. If the magnitudes of the horizontal and vertical components of the resultant hydrostatic force are known, the tangent of the angle the resultant hydrostatic force makes with the horizontal is

\[
\tan \alpha = \frac{F_y}{F_R}
\]

**Discussion**

This fact makes analysis of circular-shaped surfaces simple. There is no corresponding simplification for shapes other than circular, unfortunately.

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3-67

**Solution**

A car is submerged in water. The hydrostatic force on the door and its line of action are to be determined for the cases of the car containing atmospheric air and the car is filled with water.

**Assumptions**

1. The bottom surface of the lake is horizontal.
2. The door can be approximated as a vertical rectangular plate.
3. The pressure in the car remains at atmospheric value since there is no water leaking in, and thus no compression of the air inside. Therefore, we can ignore the atmospheric pressure in calculations since it acts on both sides of the door.

**Properties**

We take the density of lake water to be 1000 kg/m³ throughout.

**Analysis**

(a) When the car is well-sealed and thus the pressure inside the car is the atmospheric pressure, the average pressure on the outer surface of the door is the pressure at the centroid (midpoint) of the surface, and is determined to be

\[
P_{ave} = P_C = \rho gh_C = \rho g(s + b/2)
\]

\[
= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(10 + 1.1/2 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)
\]

\[
= 103.5 \text{ kN/m}^2
\]

Then the resultant hydrostatic force on the door becomes

\[
F_R = P_{ave} A = (103.5 \text{ kN/m}^2)(0.9 \text{ m} \times 1.1 \text{ m}) = 102.5 \text{ kN}
\]

The pressure center is directly under the midpoint of the plate, and its distance from the surface of the lake is determined to be

\[
y_p = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 10 + \frac{1.1}{2} + \frac{1.1^2}{12(10 + 1.1/2)} = 10.56 \text{ m}
\]

(b) When the car is filled with water, the net force normal to the surface of the door is zero since the pressure on both sides of the door will be the same.

**Discussion**

Note that it is impossible for a person to open the door of the car when it is filled with atmospheric air. But it takes little effort to open the door when car is filled with water, because then the pressure on each side of the door is the same.
Solution  The height of a water reservoir is controlled by a cylindrical gate hinged to the reservoir. The hydrostatic force on the cylinder and the weight of the cylinder per ft length are to be determined.

Assumptions  1 The hinge is frictionless.  2 Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.

Properties  We take the density of water to be 62.4 lbm/ft³ throughout.

Analysis  

(a) We consider the free body diagram of the liquid block enclosed by the circular surface of the cylinder and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block per ft length of the cylinder are:

Horizontal force on vertical surface:

\[ F_h = P_{av} A = \rho g h_c A = \rho g (s + R/2) A \]
\[ = (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(13 + 2/2 \text{ ft})(2 \text{ ft} \times 1 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) \]
\[ = 1747 \text{ lbf} \]

Vertical force on horizontal surface (upward):

\[ F_y = P_{av} A = \rho g h_c A = \rho g h_{bottom} A \]
\[ = (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(15 \text{ ft})(2 \text{ ft} \times 1 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) \]
\[ = 1872 \text{ lbf} \]

Weight of fluid block per ft length (downward):

\[ W = mg = \rho g V = \rho g (R^2 - \pi R^2/4)(1 \text{ ft}) = \rho g R^2 (1 - \pi/4)(1 \text{ ft}) \]
\[ = (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(2 \text{ ft}^2)(1 - \pi/4)(1 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) \]
\[ = 54 \text{ lbf} \]

Therefore, the net upward vertical force is

\[ F_y = F_y - W = 1872 - 54 = 1818 \text{ lbf} \]

Then the magnitude and direction of the hydrostatic force acting on the cylindrical surface become

\[ F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{1747^2 + 1818^2} = 2521 \text{ lbf} \approx 2520 \text{ lbf} \]

\[ \tan \theta = \frac{F_y}{F_h} = \frac{1818 \text{ lbf}}{1747 \text{ lbf}} = 1.041 \rightarrow \theta = 46.1° \]

Therefore, the magnitude of the hydrostatic force acting on the cylinder is 2521 lbf per ft length of the cylinder, and its line of action passes through the center of the cylinder making an angle 46.1° upwards from the horizontal.

(b) When the water level is 15-ft high, the gate opens and the reaction force at the bottom of the cylinder becomes zero. Then the forces other than those at the hinge acting on the cylinder are its weight, acting through the center, and the hydrostatic force exerted by water. Taking a moment about the point A where the hinge is and equating it to zero gives

\[ F_R R \sin \theta - W_{cy} R = 0 \rightarrow W_{cy} = F_R \sin \theta = (2521 \text{ lbf})\sin 46.1° = 1817 \text{ lbf} \approx 1820 \text{ lbf} \text{ (per ft)} \]

Discussion  The weight of the cylinder per ft length is determined to be 1820 lbf, which corresponds to a mass of 1820 lbm, and to a density of 145 lbm/ft³ for the material of the cylinder.
3-43

Chapter 3 Pressure and Fluid Statics

Solution An above the ground swimming pool is filled with water. The hydrostatic force on each wall and the distance of the line of action from the ground are to be determined, and the effect of doubling the wall height on the hydrostatic force is to be assessed.

Assumptions Atmospheric pressure acts on both sides of the wall of the pool, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m³ throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

\[ P_{avg} = P_c = \rho g h_c = \rho g \left( \frac{h}{2} \right) \]

\[ = (1000 \, \text{kg/m}^3)(9.81 \, \text{m/s}^2)\left( \frac{1 \, \text{N}}{1 \, \text{kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \, \text{m}}{1 \, \text{kg} \cdot \text{m/s}^2} \right) \]

\[ = 7357.5 \, \text{N/m}^2 \]

Then the resultant hydrostatic force on each wall becomes

\[ F_R = P_{avg} A = (7357.5 \, \text{N/m}^2)(4 \, \text{m} \times 1.5 \, \text{m}) = 44,145 \, \text{N} \approx 44.1 \, \text{kN} \]

The line of action of the force passes through the pressure center, which is \( \frac{2h}{3} \) from the free surface and \( \frac{h}{3} \) from the bottom of the pool. Therefore, the distance of the line of action from the ground is

\[ y_p = \frac{h}{3} = \frac{1.5}{3} = 0.50 \, \text{m} \] (from the bottom)

If the height of the walls of the pool is doubled, the hydrostatic force quadruples since

\[ F_R = \rho g h_c A = \rho g \left( \frac{h}{2} \right)(h \times w) = \rho g w h^2 / 2 \]

and thus the hydrostatic force is proportional to the square of the wall height, \( h^2 \).

Discussion This is one reason why above-ground swimming pools are not very deep, whereas in-ground swimming pools can be quite deep.
Solution  A dam is filled to capacity. The total hydrostatic force on the dam, and the pressures at the top and the bottom are to be determined.

Assumptions  Atmospheric pressure acts on both sides of the dam, and thus it can be ignored in calculations for convenience.

Properties  We take the density of water to be 62.4 lbm/ft³ throughout.

Analysis  The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

\[
\bar{P}_\text{avg} = \rho g h_c = \rho g \left( \frac{h}{2} \right) = \left( 62.4 \text{ lbm/ft}^3 \right) \left( 32.2 \text{ ft/s}^2 \right) \left( \frac{200}{2} \text{ ft} \right) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) = 6240 \frac{\text{lbf}}{\text{ft}^2}
\]

Then the resultant hydrostatic force acting on the dam becomes

\[
F_R = \bar{P}_\text{avg} A = (6240 \frac{\text{lbf}}{\text{ft}^2}) (200 \text{ ft} \times 1200 \text{ ft}) = 1.50 \times 10^8 \text{ lbf}
\]

Resultant force per unit area is pressure, and its value at the top and the bottom of the dam becomes

\[
P_{\text{top}} = \rho g h_{\text{top}} = 0 \frac{\text{lbf}}{\text{ft}^2}
\]

\[
P_{\text{bottom}} = \rho g h_{\text{bottom}} = \left( 62.4 \text{ lbm/ft}^3 \right) \left( 32.2 \text{ ft/s}^2 \right) (200 \text{ ft}) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) = 12,480 \frac{\text{lbf}}{\text{ft}^2} = 12,500 \frac{\text{lbf}}{\text{ft}^2}
\]

Discussion  The values above are gave pressures, of course. The gage pressure at the bottom of the dam is about 86.6 psig, or 101.4 psia, which is almost seven times greater than standard atmospheric pressure.
Solution  A room in the lower level of a cruise ship is considered. The hydrostatic force acting on the window and the pressure center are to be determined.

Assumptions  Atmospheric pressure acts on both sides of the window, and thus it can be ignored in calculations for convenience.

Properties  The specific gravity of sea water is given to be 1.025, and thus its density is 1025 kg/m³.

Analysis  The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

\[ P_{ave} = P_C = \rho gh_c = (1025 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(4 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 40,221 \text{ N/m}^2 \]

Then the resultant hydrostatic force on each wall becomes

\[ F_R = P_{ave} A = P_{ave} \left( \frac{\pi D^2}{4} \right) \]
\[ = (40,221 \text{ N/m}^2) \left( \frac{\pi (0.3 \text{ m})^2}{4} \right) \approx 2843 \text{ N} \approx 2840 \text{ (three significant digit)} \]

The line of action of the force passes through the pressure center, whose vertical distance from the free surface is determined from

\[ y_P = y_c + \frac{I_{xx,c}}{y_c A} = y_c + \frac{\pi R^4}{4 y_c} \]
\[ = y_c + \frac{R^2}{4 y_c} \]
\[ = 4 + \frac{(0.15 \text{ m})^2}{4(5 \text{ m})} = 4.001 \text{ m} \]

Discussion  For small surfaces deep in a liquid, the pressure center nearly coincides with the centroid of the surface. Here, in fact, to three significant digits in the final answer, the center of pressure and centroid are coincident. We give the answer to four significant digits to show that the center of pressure and the centroid are not coincident.
**Solution**

The cross-section of a dam is a quarter-circle. The hydrostatic force on the dam and its line of action are to be determined.

**Assumptions**

Atmospheric pressure acts on both sides of the dam, and thus it can be ignored in calculations for convenience.

**Properties**

We take the density of water to be 1000 kg/m³ throughout.

**Analysis**

We consider the free body diagram of the liquid block enclosed by the circular surface of the dam and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are:

Horizontal force on vertical surface:

\[
F_H = F_v = P_{avg} A = \rho g h_c A = \rho g \left( \frac{R}{2} \right) A
\]

\[
= (1000 \text{ kg/m}^3) (9.81 \text{ m/s}^2) \left( \frac{10 \text{ m}}{2} \right) (10 \text{ m} \times 100 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right)
\]

\[
= 4.905 \times 10^7 \text{ N}
\]

Vertical force on horizontal surface is zero since it coincides with the free surface of water. The weight of fluid block per m length

\[
F_V = W = \rho g V = \rho g \left[ \frac{W}{m} \times \pi R^2 / 4 \right]
\]

\[
= (1000 \text{ kg/m}^3) (9.81 \text{ m/s}^2) \left( \frac{100 \text{ m}}{4} \right) (10 \text{ m})^2 / \pi \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right)
\]

\[
= 7.705 \times 10^7 \text{ N}
\]

Then the magnitude and direction of the hydrostatic force acting on the surface of the dam become

\[
F_H = \sqrt{F_H^2 + F_V^2} = \sqrt{(4.905 \times 10^7 \text{ N})^2 + (7.705 \times 10^7 \text{ N})^2} \approx 9.134 \times 10^7 \text{ N} \approx 9.13 \times 10^7 \text{ N}
\]

\[
\tan \theta = \frac{F_V}{F_H} = \frac{7.705 \times 10^7 \text{ N}}{4.905 \times 10^7 \text{ N}} = 1.571 \quad \rightarrow \quad \theta = 57.5^\circ
\]

Therefore, the line of action of the hydrostatic force passes through the center of the curvature of the dam, making 57.5° downwards from the horizontal.

**Discussion**

If the shape were not circular, it would be more difficult to determine the line of action.
Solution

A rectangular plate hinged about a horizontal axis along its upper edge blocks a fresh water channel. The plate is restrained from opening by a fixed ridge at a point $B$. The force exerted to the plate by the ridge is to be determined.

**Assumptions**

Atmospheric pressure acts on both sides of the plate, and thus it can be ignored in calculations for convenience.

**Properties**

We take the density of water to be 1000 kg/m$^3$ throughout.

**Analysis**

The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{ave} = P_C = \rho gh = \rho \left(\frac{h}{2}\right)$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5/2 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right) = 24.53 \text{ kN/m}^2$$

Then the resultant hydrostatic force on each wall becomes

$$F_R = P_{ave}A = (24.53 \text{ kN/m}^2)(6 \text{ m} \times 5 \text{ m}) = 735.9 \text{ m}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface,

$$y_p = \frac{2h}{3} = \frac{2 \times (5 \text{ m})}{3} = 3.333 \text{ m}$$

Taking the moment about point $A$ and setting it equal to zero gives

$$\sum M_A = 0 \rightarrow F_R(s + y_p) = F_{ridge}AB$$

Solving for $F_{ridge}$ and substituting, the reaction force is determined to be

$$F_{ridge} = \frac{s + y_p}{AB} F_R = \frac{(1 + 3.333) \text{ m}}{5 \text{ m}} (735.9 \text{ kN}) = 638 \text{ kN}$$

**Discussion**

The difference between $F_R$ and $F_{ridge}$ is the force acting on the hinge at point $A$. 

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Solution  The previous problem is reconsidered. The effect of water depth on the force exerted on the plate by the ridge as the water depth varies from 0 to 5 m in increments of 0.5 m is to be investigated.

Analysis  The EES Equations window is printed below, followed by the tabulated and plotted results.

\[
g=9.81 \text{ m/s}^2 \\
rho=1000 \text{ kg/m}^3 \\
s=1 \text{ m} \\
w=5 \text{ m} \\
A=w\times h \\
P_{\text{ave}}=\rho g h/2000 \text{ kPa} \\
F_R=P_{\text{ave}} A \text{ kN} \\
y_p=2h/3 \\
F_{\text{ridge}}=(s+y_p)F_R/(s+h)
\]

<table>
<thead>
<tr>
<th>Dept h, m</th>
<th>(P_{\text{ave}}, \text{kPa})</th>
<th>(F_R, \text{kN})</th>
<th>(y_p, \text{m})</th>
<th>(F_{\text{ridge}}, \text{kN})</th>
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<td>24.53</td>
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Discussion  The force on the ridge does not increase linearly, as we may have suspected.
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3-76E

**Solution**  
The flow of water from a reservoir is controlled by an L-shaped gate hinged at a point $A$. The required weight $W$ for the gate to open at a specified water height is to be determined.

**Assumptions** 1 Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. 2 The weight of the gate is negligible.

**Properties**  
We take the density of water to be 62.4 lbm/ft$^3$ throughout.

**Analysis**  
The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{avg} = \rho gh_c = \rho g \left( h / 2 \right)$$

$$= \left( 62.4 \text{ lbm/ft}^3 \right) \left( 32.2 \text{ ft/s}^2 \right) \left( 8 / 2 \text{ ft} \right) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right)$$

$$= 249.6 \text{ lbf/ft}^2$$

Then the resultant hydrostatic force acting on the dam becomes

$$F_R = P_{avg} A = \left( 249.6 \text{ lbf/ft}^2 \right) (8 \text{ ft} \times 5 \text{ ft}) = 9984 \text{ lbf}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface,

$$y_p = \frac{2h}{3} = \frac{2 \times (8 \text{ ft})}{3} = 5.333 \text{ ft}$$

Taking the moment about point $A$ and setting it equal to zero gives

$$\sum M_A = 0 \rightarrow F_R (s + y_p) = W AB$$

Solving for $W$ and substituting, the required weight is determined to be

$$W = \frac{s + y_p}{AB} F_R = \frac{\left( 7 + 5.333 \right) \text{ ft}}{8 \text{ ft}} \left( 9984 \text{ lbf} \right) = 15,390 \text{ lbf} \approx 15,400 \text{ lbf}$$

**Discussion**  
Note that the required weight is inversely proportional to the distance of the weight from the hinge.
Solution Two parts of a water trough of semi-circular cross-section are held together by cables placed along the length of the trough. The tension $T$ in each cable when the trough is full is to be determined.

Assumptions 1 Atmospheric pressure acts on both sides of the trough wall, and thus it can be ignored in calculations for convenience. 2 The weight of the trough is negligible.

Properties We take the density of water to be 1000 kg/m$^3$ throughout.

Analysis To expose the cable tension, we consider half of the trough whose cross-section is quarter-circle. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are:

Horizontal force on vertical surface:

$$F_H = F_A = P_{ave}A = \rho gh_c A = \rho g (R/2) A$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.7/2 \text{ m})(0.5 \text{ m} \times 3 \text{ m}) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$= 5150 \text{ N}$$

The vertical force on the horizontal surface is zero, since it coincides with the free surface of water. The weight of fluid block per 3-m length is

$$F_V = W = \rho g V = \rho g [w \times \pi R^2 / 4]$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3 \text{ m})\pi(0.7 \text{ m})^2 / 4 \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$= 11,326 \text{ N}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the 3-m long section of the trough become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(5150 \text{ N})^2 + (11,326 \text{ N})^2} = 12,442 \text{ N}$$

$$\tan \theta = \frac{F_V}{F_H} = \frac{11,326 \text{ N}}{5150 \text{ N}} = 2.199 \quad \rightarrow \quad \theta = 65.5^\circ$$

Therefore, the line of action passes through the center of the curvature of the trough, making 65.5$^\circ$ downwards from the horizontal. Taking the moment about point $A$ where the two parts are hinged and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R R \sin(90 - 65.5)^\circ = TR$$

Solving for $T$ and substituting, the tension in the cable is determined to be

$$T = F_R \sin(90 - 65.5)^\circ = (12,442 \text{ N}) \sin(90 - 65.5)^\circ = 5160 \text{ N}$$

Discussion This problem can also be solved without finding $F_R$ by finding the lines of action of the horizontal hydrostatic force and the weight.
Solution  Two parts of a water trough of triangular cross-section are held together by cables placed along the length of the trough. The tension $T$ in each cable when the trough is filled to the rim is to be determined.

Assumptions  1 Atmospheric pressure acts on both sides of the trough wall, and thus it can be ignored in calculations for convenience. 2 The weight of the trough is negligible.

Properties  We take the density of water to be 1000 kg/m$^3$ throughout.

Analysis  To expose the cable tension, we consider half of the trough whose cross-section is triangular. The water height $h$ at the midsection of the trough and width of the free surface are

$$h = L \sin \theta = (0.75 \text{ m}) \sin 45^\circ = 0.530 \text{ m}$$

$$b = L \cos \theta = (0.75 \text{ m}) \cos 45^\circ = 0.530 \text{ m}$$

The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface:

$$F_{H} = F_{v} = P_{avg} A = \rho g h c A = \rho g \left( \frac{h}{2} \right) A$$

$$= \left( 1000 \text{ kg/m}^3 \right) \left( 9.81 \text{ m/s}^2 \right) \left( 0.530 / 2 \text{ m} \right) \left( 0.530 \text{ m} \times 6 \text{ m} \right) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right)$$

$$= 8267 \text{ N}$$

The vertical force on the horizontal surface is zero since it coincides with the free surface of water. The weight of fluid block per 6-m length is

$$F_{V} = W = \rho g V = \rho g [w \times bh / 2]$$

$$= \left( 1000 \text{ kg/m}^3 \right) \left( 9.81 \text{ m/s}^2 \right) \left( 6 \text{ m} \right) \left( 0.530 \text{ m} \right) \left( 0.530 \text{ m} \right) / 2 \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right)$$

$$= 8267 \text{ N}$$

The distance of the centroid of a triangle from a side is $1/3$ of the height of the triangle for that side. Taking the moment about point $A$ where the two parts are hinged and setting it equal to zero gives

$$\sum M_{A} = 0 \quad \rightarrow \quad W \frac{b}{3} + F_{H} \frac{h}{3} = Th$$

Solving for $T$ and substituting, and noting that $h = b$, the tension in the cable is determined to be

$$T = \frac{F_{H} + W}{3} = \left( \frac{8267 + 8267}{3} \right) \text{ N} = 5511 \text{ N} \cong 5510 \text{ N}$$

Discussion  The analysis is simplified because of the symmetry of the trough.
Solution  Two parts of a water trough of triangular cross-section are held together by cables placed along the length of the trough. The tension $T$ in each cable when the trough is filled to the rim is to be determined.

Assumptions  1. Atmospheric pressure acts on both sides of the trough wall, and thus it can be ignored in calculations for convenience. 2. The weight of the trough is negligible.

Properties  We take the density of water to be 1000 kg/m$^3$ throughout.

Analysis  To expose the cable tension, we consider half of the trough whose cross-section is triangular. The water height is given to be $h = 0.4$ m at the midsection of the trough, which is equivalent to the width of the free surface $b$ since $\tan 45^\circ = b/h = 1$. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface:

$$F_h = F_s = P_{avg}A = \rho gh \cdot A = \rho g \left( \frac{h}{2} \right) A$$

$$= \left(1000 \text{ kg/m}^3 \right) \left(9.81 \text{ m/s}^2 \right) \left(0.4 / 2 \text{ m} \right) \left(0.4 \text{ m} \times 3 \text{ m} \right) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right)$$

$$= 2354 \text{ N}$$

The vertical force on the horizontal surface is zero since it coincides with the free surface of water. The weight of fluid block per 3-m length is

$$F_y = W = \rho g V = \rho g \left[\frac{w \times bh}{2} \right]$$

$$= \left(1000 \text{ kg/m}^3 \right) \left(9.81 \text{ m/s}^2 \right) \left[(3 \text{ m})(0.4 \text{ m})(0.4 \text{ m})/2 \right] \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right)$$

$$= 2354 \text{ N}$$

The distance of the centroid of a triangle from a side is 1/3 of the height of the triangle for that side. Taking the moment about point $A$ where the two parts are hinged and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad W \frac{b}{3} + F_h \frac{h}{3} = Th$$

Solving for $T$ and substituting, and noting that $h = b$, the tension in the cable is determined to be

$$T = \frac{F_h + W}{3} = \frac{1}{3} \left(2354 + 2354\right) \text{ N} = 1569 \text{ N} \approx 1570 \text{ N}$$

Discussion  The tension force here is a factor of about 3.5 smaller than that of the previous problem, even though the trough is more than half full.
Chapter 3 Pressure and Fluid Statics

Solution A retaining wall against mud slide is to be constructed by rectangular concrete blocks. The mud height at which the blocks will start sliding, and the blocks will tip over are to be determined.

Assumptions Atmospheric pressure acts on both sides of the wall, and thus it can be ignored in calculations for convenience.

Properties The density is given to be 1800 kg/m³ for the mud, and 2700 kg/m³ for concrete blocks.

Analysis (a) The weight of the concrete wall per unit length \( L = 1 \text{ m} \) and the friction force between the wall and the ground are

\[
W_{\text{block}} = \rho g V = (2700 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.25 \times 1.2 \times 1 \text{ m}^3) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 7946 \text{ N}
\]

\[
F_{\text{friction}} = \mu W_{\text{block}} = 0.3(7946 \text{ N}) = 2384 \text{ N}
\]

The hydrostatic force exerted by the mud to the wall is

\[
F_H = F_s = P_{\text{avg}} A = \rho g h_c A = \rho g \left( \frac{h}{2} \right) A
\]

\[
= (1800 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\left( \frac{h}{2} \right)(1 \times h) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right)
\]

\[
= 8829h^2 \text{ N}
\]

Setting the hydrostatic and friction forces equal to each other gives

\[
F_H = F_{\text{friction}} \rightarrow 8829h^2 = 2384 \rightarrow h = 0.27 \text{ m}
\]

(b) The line of action of the hydrostatic force passes through the pressure center, which is \(2h/3\) from the free surface. The line of action of the weight of the wall passes through the midplane of the wall. Taking the moment about point \(A\) and setting it equal to zero gives

\[
\sum M_A = 0 \rightarrow W_{\text{block}}(t/2) = F_H(h/3) \rightarrow W_{\text{block}}(t/2) = 8829h^3 / 3
\]

Solving for \(h\) and substituting, the mud height for tip over is determined to be

\[
h = \left( \frac{3W_{\text{block}}t}{2 \times 8829} \right)^{1/3} = \left( \frac{3 \times 7946 \times 0.25}{2 \times 8829} \right)^{1/3} = 0.70 \text{ m}
\]

Discussion The concrete wall will slide before tipping. Therefore, sliding is more critical than tipping in this case.
Solution A retaining wall against mud slide is to be constructed by rectangular concrete blocks. The mud height at which the blocks will start sliding, and the blocks will tip over are to be determined.

Assumptions Atmospheric pressure acts on both sides of the wall, and thus it can be ignored in calculations for convenience.

Properties The density is given to be 1800 kg/m$^3$ for the mud, and 2700 kg/m$^3$ for concrete blocks.

Analysis

(a) The weight of the concrete wall per unit length ($L = 1$ m) and the friction force between the wall and the ground are

$$W_{\text{block}} = \rho g V = (2700 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.4 \times 0.8 \times 1 \text{ m}^3)\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 8476 \text{ N}$$

$$F_{\text{friction}} = \mu W_{\text{block}} = 0.3(8476 \text{ N}) = 2543 \text{ N}$$

The hydrostatic force exerted by the mud to the wall is

$$F_H = F_v = P_{\text{avg}} A = \rho gh_c A = \rho g \left(\frac{h}{2}\right) A$$

$$= (1800 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\left(\frac{h}{2}\right)(1 \times h)\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$= 8829h^2 \text{ N}$$

Setting the hydrostatic and friction forces equal to each other gives

$$F_H = F_{\text{friction}} \rightarrow 8829 h^2 = 2543 \rightarrow h = 0.54 \text{ m}$$

(b) The line of action of the hydrostatic force passes through the pressure center, which is $2h/3$ from the free surface. The line of action of the weight of the wall passes through the midplane of the wall. Taking the moment about point $A$ and setting it equal to zero gives

$$\sum M_A = 0 \rightarrow W_{\text{block}}(t/2) = F_H (h/3) \rightarrow W_{\text{block}}(t/2) = 8829h^3 / 3$$

Solving for $h$ and substituting, the mud height for tip over is determined to be

$$h = \left(\frac{3W_{\text{block}}t}{2 \times 8829}\right)^{1/3} = \left(\frac{3 \times 8476 \times 0.3}{2 \times 8829}\right)^{1/3} = 0.76 \text{ m}$$

Discussion Note that the concrete wall will slide before tipping. Therefore, sliding is more critical than tipping in this case.
**Solution** A quarter-circular gate hinged about its upper edge controls the flow of water over the ledge at B where the gate is pressed by a spring. The minimum spring force required to keep the gate closed when the water level rises to A at the upper edge of the gate is to be determined.

**Assumptions** 1 The hinge is frictionless. 2 Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. 3 The weight of the gate is negligible.

**Properties** We take the density of water to be 1000 kg/m³ throughout.

**Analysis** We consider the free body diagram of the liquid block enclosed by the circular surface of the gate and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

- **Horizontal force on vertical surface:**
  \[ F_H = F_x = P_{avg}A = \rho g h_c A = \rho g (R/2)A \]
  \[ = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3/2 \text{ m})(4 \text{ m} \times 3 \text{ m}) \left( \frac{1 \text{kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \]
  \[ = 176.6 \text{ kN} \]

- **Vertical force on horizontal surface (upward):**
  \[ F_y = P_{avg}A = \rho g h_c A = \rho g h_{motion} A \]
  \[ = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3 \text{ m})(4 \text{ m} \times 3 \text{ m}) \left( \frac{1 \text{kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 353.2 \text{ kN} \]

- **The weight of fluid block per 4-m length (downwards):**
  \[ W = \rho g V = \rho g \left[ w \times \pi R^2 / 4 \right] \]
  \[ = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \left[ (4 \text{ m}) \pi (3 \text{ m})^2 / 4 \right] \left( \frac{1 \text{kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 277.4 \text{ kN} \]

Therefore, the net upward vertical force is

\[ F_y = F_y - W = 353.2 - 277.4 = 75.8 \text{ kN} \]

Then the magnitude and direction of the hydrostatic force acting on the surface of the 4-m long quarter-circular section of the gate become

\[ F_R = \sqrt{F_{H}^2 + F_{V}^2} = \sqrt{(176.6 \text{ kN})^2 + (75.8 \text{ kN})^2} = 192.2 \text{ kN} \]

\[ \tan \theta = \frac{F_{V}}{F_{H}} = \frac{75.8 \text{ kN}}{176.6 \text{ kN}} = 0.429 \rightarrow \theta = 23.2° \]

Therefore, the magnitude of the hydrostatic force acting on the gate is 192.2 kN, and its line of action passes through the center of the quarter-circular gate making an angle 23.2° upwards from the horizontal.

The minimum spring force needed is determined by taking a moment about the point A where the hinge is, and setting it equal to zero,

\[ \sum M_A = 0 \rightarrow F_R R \sin(90° - \theta) - F_{spring} R = 0 \]

Solving for \( F_{spring} \) and substituting, the spring force is determined to be

\[ F_{spring} = F_y \sin(90° - \theta) = (192.2 \text{ kN}) \sin(90° - 23.2°) = 177 \text{ kN} \]

**Discussion** Several variations of this design are possible. Can you think of some of them?
Solution
A quarter-circular gate hinged about its upper edge controls the flow of water over the ledge at \( B \) where the gate is pressed by a spring. The minimum spring force required to keep the gate closed when the water level rises to \( A \) at the upper edge of the gate is to be determined.

Assumptions
1. The hinge is frictionless.
2. Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.
3. The weight of the gate is negligible.

Properties
We take the density of water to be 1000 \( \text{kg/m}^3 \) throughout.

Analysis
We consider the free body diagram of the liquid block enclosed by the circular surface of the gate and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

**Horizontal force on vertical surface**:
\[
F_H = F_x = P_{\text{ave}}A = \rho gh_c A = \rho g (R/2) A
\]
\[
= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(4/2 \text{ m})(4 \text{ m} \times 4 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)
\]
\[
= 313.9 \text{ kN}
\]

**Vertical force on horizontal surface** (upward):
\[
F_y = P_{\text{ave}}A = \rho gh_c A = \rho g h_{\text{bottom}} A
\]
\[
= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(4 \text{ m})(4 \text{ m} \times 4 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)
\]
\[
= 627.8 \text{ kN}
\]

The weight of fluid block per 4-m length (downwards):
\[
W = \rho g V = \rho g [\pi R^2 / 4]
\]
\[
= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(4 \text{ m})(4 \text{ m})/4]\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)
\]
\[
= 493.1 \text{ kN}
\]

Therefore, the net upward vertical force is
\[
F_y = F_x - W = 627.8 - 493.1 = 134.7 \text{ kN}
\]

Then the magnitude and direction of the hydrostatic force acting on the surface of the 4-m long quarter-circular section of the gate become
\[
F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(313.9 \text{ kN})^2 + (134.7 \text{ kN})^2} = 341.6 \text{ kN}
\]
\[
\tan \theta = \frac{F_V}{F_H} = \frac{134.7 \text{ kN}}{313.9 \text{ kN}} = 0.429 \quad \Rightarrow \quad \theta = 23.2^\circ
\]

Therefore, the magnitude of the hydrostatic force acting on the gate is 341.6 kN, and its line of action passes through the center of the quarter-circular gate making an angle 23.2° upwards from the horizontal.

The minimum spring force needed is determined by taking a moment about the point \( A \) where the hinge is, and setting it equal to zero,
\[
\sum M_A = 0 \quad \Rightarrow \quad F_R R \sin(90^\circ - \theta) - F_{\text{spring}} R = 0
\]

Solving for \( F_{\text{spring}} \) and substituting, the spring force is determined to be
\[
F_{\text{spring}} = F_R \sin(90^\circ - \theta) = (341.6 \text{ kN})\sin(90^\circ - 23.2^\circ) = 314 \text{ kN}
\]

Discussion
If the previous problem is solved using a program like EES, it is simple to repeat with different values.
Solution  We are to determine the force on the upper face of a submerged flat plate.

Assumptions  1 The water is incompressible. 2 The system is at rest during the analysis (hydrostatics). 3 Atmospheric pressure is ignored since it acts on both sides of the plate.

Analysis  

(a) At first, and as a good approximation as plate thickness \( t \) approaches zero, the pressure force is simply 

\[
F = \rho gHA = \rho gHbw,
\]

since the centroid of the plate is at its center regardless of the tilt angle. However, the plate thickness must be taken into account since we are concerned with the upper face of the plate. Some trig yields that the depth of water from the surface to the centroid of the upper plate is 

\[
H - \left(\frac{t}{2}\right)\cos\theta,
\]

i.e., somewhat smaller than \( H \) itself since the upper face of the plate is above the center of the plate when it is tilted \((\theta > 0)\). Thus,

\[
F = \rho g \left( H - \frac{t}{2} \cos\theta \right)bw
\]

(b) For the given values,

\[
F = \rho g \left( H - \frac{t}{2} \cos\theta \right)bw
= \left(998.3 \frac{\text{kg}}{\text{m}^3}\right) \left(9.807 \frac{\text{m}}{\text{s}^2}\right) \left(1.25 \text{ m} - \frac{0.200 \text{ m}}{2} \cos 30^\circ\right) (1.00 \text{ m})(1.00 \text{ m})
\]

\[
= 11390 \text{ N}
\]

Thus, the “gage” force (ignoring atmospheric pressure) on the upper plate surface is \( F = 11,400 \text{ N} \) (to three digits).

Discussion  If we ignore plate thickness (set \( t = 0 \)), the result becomes 12,200 N, which represents an error of around 7%, since the plate here is fairly thick.
Solution  We are to calculate the force on a submerged flat plate for various values of water depth $h$, and we are to compare our manual calculations to those generated using CFD.

Assumptions  1 The water is incompressible.  2 The system is at rest during the analysis (hydrostatics).  3 Atmospheric pressure is ignored since it acts on both sides of the plate.

Analysis  
(a) The total force by the water on the plate (neglecting atmospheric pressure) was calculated in the previous problem. From the equation, and since $\theta$ is constant, $F$ varies linearly with $H$. Example calculation: when $H = 4.00$ m, $w = 1.00$ m, $b = 1.00$ m, $t = 0.200$ m, $\theta = 90^\circ$, $g = 9.807$ m/s$^2$, and $\rho = 998.2$ kg/m$^3$, $F$ is calculated as

$$F = \rho g \left( H - \frac{t}{2} \cos \theta \right) bw$$

$$= \left( 998.2 \text{ kg/m}^3 \right) \left( 9.807 \text{ m/s}^2 \right) \left( 4.00 \text{ m} - \frac{0.200 \text{ m}}{2} \cos 90^\circ \right) (1.00 \text{ m})(1.00 \text{ m}) \left( \frac{N}{\text{kg} \cdot \text{m/s}^2} \right)$$

$$= 39157.4 \text{ N}$$

(b) The FlowLab template was run with the above values, but with $1.00 < H < 10.0$ m. The results are compared with the manual calculations in the table and plot shown below. Note: FlowLab results include atmospheric pressure, so we subtract $P_{\text{atm}}A = (101,325 \text{ N/m}^2)(1.00 \text{ m}^2) = 101,325 \text{ N}$ from the FlowLab values for force on the plate.

<table>
<thead>
<tr>
<th>$H$ (m)</th>
<th>Manual calculation</th>
<th>FlowLab calculation</th>
<th>FlowLab calculation</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9789.3474</td>
<td>111117</td>
<td>9792</td>
<td>0.0271</td>
</tr>
<tr>
<td>2</td>
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<td>120910</td>
<td>19585</td>
<td>0.0322</td>
</tr>
<tr>
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<td>6</td>
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<td>0.0302</td>
</tr>
</tbody>
</table>

The agreement between the manual and CFD calculations is excellent as expected since this is a simple hydrostatics problem. The percentage difference between the manual and CFD calculations is less than 0.04% for all the cases. We also verify on the plot that the relationship between $F$ and $H$ is indeed linear.

Discussion  The force increases as $H$ increases because of hydrostatic pressure head on the plate.
Solution  We are to calculate the force on a submerged flat plate for various values of angle $\theta$ at a fixed water depth $h$, and we are to compare our manual calculations to those generated using CFD.

Assumptions  1 The water is incompressible. 2 The system is at rest during the analysis (hydrostatics). 3 Atmospheric pressure is ignored since it acts on both sides of the plate.

Analysis  (a) The force of the water on the plate (neglecting atmospheric pressure) was calculated in a previous problem. From the equation, with constant $H$, $F$ varies nonlinearly with $\theta$ due to the cosine. Sample calculation: when $H = 4.00$ m, $w = 1.25$ m, $b = 1.00$ m, $t = 0.200$ m, $\theta = 20^\circ$, $g = 9.807$ m/s$^2$, and $\rho = 998.2$ kg/m$^3$, $F$ is calculated as

$$ F = \rho g \left( H - \frac{t}{2} \cos \theta \right) bw $$

$$ = \left( 998.2 \frac{\text{kg}}{\text{m}^3} \right) \left( 9.807 \frac{\text{m}^3}{\text{s}^2} \right) \left( 1.25 \text{ m} - \frac{0.200 \text{ m}}{2} \cos 20^\circ \right) (1.00 \text{ m})(1.00 \text{ m}) \left( \frac{\text{N}}{\text{kg} \cdot \text{m/s}^2} \right) $$

$$ = 11316.8 \text{ N} $$

(b) The FlowLab template was run with the above values, but with $0 < \theta < 90^\circ$. The results are compared with the manual calculations in the table and plot shown below. Note: FlowLab results include atmospheric pressure, so we subtract $P_{\text{atm}}A = (101,325 \text{ N/m}^2)(1.00 \text{ m}^2) = 101,325 \text{ N}$ from the FlowLab values for force on the plate.

<table>
<thead>
<tr>
<th>$\theta$ (°)</th>
<th>$\theta$ (rad)</th>
<th>Manual calculation</th>
<th>FlowLab calculation</th>
<th>FlowLab calculation</th>
<th>FlowLab calculation</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>11257.74951</td>
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<td>90</td>
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<td>113565</td>
<td>12240</td>
<td>0.0271</td>
<td></td>
</tr>
</tbody>
</table>

The agreement between the manual and CFD calculations is excellent as expected since this is a simple hydrostatics problem. The percentage difference between the manual and CFD calculations is less than 0.04% for all the cases. We also verify on the plot that the relationship between $F$ and $h$ is indeed nonlinear.

Discussion  The hydrostatic pressure head on the plate is not constant as it rotates because it rotates about its center, not about the center of the left face, and the plate is of finite thickness.
Solution  We are to determine the net force on a partially submerged cylindrical gate.

Assumptions  1 The water is incompressible. 2 The system is at rest during the analysis (hydrostatics). 3 Atmospheric pressure is ignored since it acts on both sides of the cylinder.

Analysis  (a) The horizontal component of the force is

\[ F_h = F_s = P_{av \text{g}} A_{\text{vertical}} = \rho g h_c A_{\text{vertical}}, \]

where \( h_c \) is the water depth from the surface to the midpoint (center) of the cylinder’s projected vertical face. Here, \( h_c = h - R/2 \). Area \( A_{\text{vertical}} \) is the area of the cylinder’s projected vertical face, which is \( Rw \). Thus,

\[ F_h = \rho g \left( h - \frac{R}{2} \right) Rw \]

As discussed in the text, the vertical component of force is composed of two parts: \( F_y \) is the vertical pressure force on the cylinder’s projected lower horizontal face, which is

\[ F_y = P_{av \text{g}} A_{\text{horizontal}} = \rho g h w \]

\( W \) is the weight of the volume of water enclosed by the curved surface and the projected horizontal and vertical faces, which is

\[ W = mg = \rho g V = \rho g \left( R^2 - \frac{\pi R^2}{4} \right) w = \rho g w R^2 \left( 1 - \frac{\pi}{4} \right). \]

Since the horizontal surface is below the volume of water calculated here, \( F_y = F_y - W \), and thus,

\[ F_v = F_y - W = \rho g w h R - \rho g w R^2 \left( 1 - \frac{\pi}{4} \right) = \rho g w \left( hR - R^2 \left( 1 - \frac{\pi}{4} \right) \right) \]

Finally, the total force by the water on the cylinder (neglecting atmospheric pressure) is

\[ F = \sqrt{F_h^2 + F_v^2} \]

or,

\[ F = \rho g w \sqrt{R^2 \left( h - \frac{R}{2} \right)^2 + \left( hR - R^2 \left( 1 - \frac{\pi}{4} \right) \right)^2} \]

(b) For the given values,

\[ F = \rho g w \sqrt{R^2 \left( h - \frac{R}{2} \right)^2 + \left( hR - R^2 \left( 1 - \frac{\pi}{4} \right) \right)^2} \]

\[ = \left( 998.3 \frac{\text{kg}}{\text{m}^3} \right) \left( 9.807 \frac{\text{m}}{\text{s}^2} \right) (1.00 \text{ m}) \times \]

\[ \sqrt{(0.500 \text{ m})^2 \left( 5.00 \text{ m} - \frac{0.500 \text{ m}}{2} \right)^2 + \left( (5.00 \text{ m})(0.500 \text{ m}) - (0.500 \text{ m})^2 \left( 1 - \frac{\pi}{4} \right) \right)^2} \left( \frac{\text{N}}{\text{kg} \cdot \text{m/s}^2} \right) \]

\[ = 33380.9 \text{ N} \]

Thus, the “gage” force (ignoring atmospheric pressure) on the cylinder surface is \( F = 33,400 \text{ N} \) (to three digits).

Discussion  There is also a moment associated with this force. If large enough, the water pressure causes the cylindrical gate to open.
Chapter 3 Pressure and Fluid Statics

Solution We are to calculate the force on a cylindrical gate for various values of water depth $h$, and we are to compare our manual calculations to those generated using CFD.

Assumptions 1 The water is incompressible. 2 The system is at rest during the analysis (hydrostatics). 3 Atmospheric pressure is ignored since it acts on both sides of the cylinder.

Analysis (a) The total force by the water on the cylinder (neglecting atmospheric pressure) was calculated in the previous problem. From the equation, $F$ is nearly linear with $h$, but not exactly, since the equation is complicated. For example, when $h = 5.00$ m, $R = 0.500$ m, $w = 1.00$ m, $g = 9.807$ m/s$^2$, and $\rho = 998.3$ kg/m$^3$, $F$ is calculated as

$$F = \rho gw \left( R^2 \left( \frac{h - R}{2} \right)^2 + \left( hR - R^2 \left( 1 - \frac{\pi}{4} \right) \right)^2 \right)$$

$$= \left( 998.3 \frac{\text{kg}}{\text{m}^3} \right) \left( 9.807 \frac{\text{m}}{\text{s}^2} \right) (1.00 \text{ m}) \times$$

$$\sqrt{(0.500 \text{ m})^2 \left( 5.00 \text{ m} - \frac{0.500 \text{ m}}{2} \right)^2 + \left( (5.00 \text{ m})(0.500 \text{ m}) - (0.500 \text{ m})^2 \right)^2 \left( 1 - \frac{\pi}{4} \right)^2 \left( \frac{\text{N}}{\text{kg} \cdot \text{m/s}^2} \right)}$$

$$= 33380.9 \text{ N}$$

(b) The FlowLab template was run with the above values, but with $0.600 < h < 5.00$ m. The results are compared with the manual calculations in the table and plot shown below.

<table>
<thead>
<tr>
<th>$h$ (m)</th>
<th>Manual calculation</th>
<th>FlowLab calculation</th>
</tr>
</thead>
<tbody>
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<td>0.6</td>
<td>2958.44916</td>
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</tr>
<tr>
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<td>5707.458266</td>
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</tr>
<tr>
<td>5</td>
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<td>33401.1</td>
</tr>
</tbody>
</table>

The agreement between the manual and CFD calculations is excellent as expected since this is a simple hydrostatics problem. We also verify on the plot that the relationship between $F$ and $h$ is indeed nearly linear.

Discussion The force increases as $h$ increases because of hydrostatic pressure head on the gate.
Solution We are to calculate the force on a cylindrical gate for various values of water width \( L \) in the reservoir, and we are to compare our manual calculations to those generated using CFD.

Assumptions 1 The water is incompressible. 2 The system is at rest during the analysis (hydrostatics). 3 Atmospheric pressure is ignored since it acts on both sides of the cylinder.

Analysis (a) The total force by the water on the cylinder (neglecting atmospheric pressure) was calculated in the previous problem. From the equation, \( F \) is not a function of \( L \). Thus, we expect to get the same \( F \) regardless of the value of \( L \). This is verified when \( h = 5.00 \text{ m}, R = 0.500 \text{ m}, w = 1.00 \text{ m}, g = 9.807 \text{ m/s}^2, \) and \( \rho = 998.3 \text{ kg/m}^3 \), which yields

\[
F = \rho w g \sqrt{R^2 \left( h - \frac{R}{2} \right)^2 + \left( hR - R^2 \left(1 - \frac{\pi}{4}\right) \right)^2}
\]

\[
= \left(998.3 \frac{\text{kg}}{\text{m}^3}\right) \left(9.807 \frac{\text{m}}{\text{s}^2}\right) (1.00 \text{ m}) \times
\left(0.500 \text{ m}^2 \left(5.00 \text{ m} - \frac{0.500 \text{ m}}{2}\right)\right)^2 + \left((5.00 \text{ m})(0.500 \text{ m}) - (0.500 \text{ m})^2 \left(1 - \frac{\pi}{4}\right)\right)^2 \left( \frac{\text{N}}{\text{kg} \cdot \text{m/s}^2} \right)
\]

\[
= 33380.9 \text{ N}
\]

(b) The FlowLab template was run with the above values, but with \( 0.200 < L < 2.00 \text{ m} \). The results are compared with the manual calculations in the table and plot shown below.

<table>
<thead>
<tr>
<th>( L (\text{m}) )</th>
<th>Manual calculation</th>
<th>FlowLab calculation</th>
</tr>
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<tbody>
<tr>
<td>0.2</td>
<td>33380.92885</td>
<td>33401.3</td>
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<tr>
<td>0.5</td>
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</tr>
<tr>
<td>2</td>
<td>33380.92885</td>
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</table>

The agreement between the manual and CFD calculations is excellent as expected since this is a simple hydrostatics problem. We also verify on the plot that the \( F \) is not a function of \( L \).

Discussion The CFD calculations are not exact, and the force differs in the first decimal place – a negligible difference.
**Buoyancy**

3-90C  
**Solution** We are to define and discuss the buoyant force.

**Analysis** The upward force a fluid exerts on an immersed body is called the buoyant force. The buoyant force is caused by the increase of pressure in a fluid with depth. The magnitude of the buoyant force acting on a submerged body whose volume is $V$ is expressed as $F_B = \rho \, g \, V$. The direction of the buoyant force is upwards, and its line of action passes through the centroid of the displaced volume.

**Discussion** If the buoyant force is greater than the body’s weight, it floats.

3-91C  
**Solution** We are to compare the buoyant force on two spheres.

**Analysis** The magnitude of the buoyant force acting on a submerged body whose volume is $V$ is expressed as $F_B = \rho \, g \, V$, which is independent of depth. Therefore, the buoyant forces acting on two identical spherical balls submerged in water at different depths is the same.

**Discussion** Buoyant force depends only on the volume of the object, not its density.

3-92C  
**Solution** We are to compare the buoyant force on two spheres.

**Analysis** The magnitude of the buoyant force acting on a submerged body whose volume is $V$ is expressed as $F_B = \rho \, g \, V$, which is independent of the density of the body ($\rho_f$ is the fluid density). Therefore, the buoyant forces acting on the 5-cm diameter aluminum and iron balls submerged in water is the same.

**Discussion** Buoyant force depends only on the volume of the object, not its density.

3-93C  
**Solution** We are to compare the buoyant forces on a cube and a sphere.

**Analysis** The magnitude of the buoyant force acting on a submerged body whose volume is $V$ is expressed as $F_B = \rho \, g \, V$, which is independent of the shape of the body. Therefore, the buoyant forces acting on the cube and sphere made of copper submerged in water are the same since they have the same volume.

**Discussion** The two objects have the same volume because they have the same mass and density.
3-94C

Solution  We are to discuss the stability of a submerged and a floating body.

Analysis  A submerged body whose center of gravity \( G \) is above the center of buoyancy \( B \), which is the centroid of the displaced volume, is unstable. But a floating body may still be stable when \( G \) is above \( B \) since the centroid of the displaced volume shifts to the side to a point \( B' \) during a rotational disturbance while the center of gravity \( G \) of the body remains unchanged. If the point \( B' \) is sufficiently far, these two forces create a restoring moment, and return the body to the original position.

Discussion  Stability analysis like this is critical in the design of ship hulls, so that they are least likely to capsize.

3-95

Solution  The density of a liquid is to be determined by a hydrometer by establishing division marks in water and in the liquid, and measuring the distance between these marks.

Properties  We take the density of pure water to be 1000 kg/m\(^3\).

Analysis  A hydrometer floating in water is in static equilibrium, and the buoyant force \( F_B \) exerted by the liquid must always be equal to the weight \( W \) of the hydrometer, \( F_B = W \).

\[
F_B = \rho g A h \quad \Rightarrow \quad W = \rho g A h
\]

where \( h \) is the height of the submerged portion of the hydrometer and \( A \) is the cross-sectional area which is constant.

In pure water: \( W = \rho_w g h_w A \)

In the liquid: \( W = \rho_{\text{liquid}} g h_{\text{liquid}} A \)

Setting the relations above equal to each other (since both equal the weight of the hydrometer) gives

\[
\rho_w g h_w A = \rho_{\text{liquid}} g h_{\text{liquid}} A
\]

Solving for the liquid density and substituting,

\[
\rho_{\text{liquid}} = \frac{h_w}{h_{\text{liquid}}} \rho_{\text{water}} = \frac{13 \text{ cm}}{(13 - 0.6) \text{ cm}} (1000 \text{ kg/m}^3) = 1048.39 \text{ kg/m}^3 \approx 1050 \text{ kg/m}^3
\]

Discussion  Note that for a given cylindrical hydrometer, the product of the fluid density and the height of the submerged portion of the hydrometer is constant in any fluid.
Section 3-96E

Solution

A concrete block is lowered into the sea. The tension in the rope is to be determined before and after the block is immersed in water.

Assumptions

1. The buoyancy force in air is negligible.
2. The weight of the rope is negligible.

Properties

The density of steel block is given to be 494 lbm/ft³.

Analysis

(a) The forces acting on the concrete block in air are its downward weight and the upward pull action (tension) by the rope. These two forces must balance each other, and thus the tension in the rope must be equal to the weight of the block:

\[ T_R = W = \rho_{\text{concrete}} g V \]

\[ = \left( 494 \text{ lbm/ft}^3 \right) \left( 32.2 \text{ ft/s}^2 \right) \left( 14.137 \text{ ft}^3 \right) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbf} \cdot \text{ft/s}^2} \right) = 6984 \text{ lbf} \]

(b) When the block is immersed in water, there is the additional force of buoyancy acting upwards. The force balance in this case gives

\[ F_B = \rho_{\text{water}} g V = \left( 62.4 \text{ lbm/ft}^3 \right) \left( 32.2 \text{ ft/s}^2 \right) \left( 14.137 \text{ ft}^3 \right) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbf} \cdot \text{ft/s}^2} \right) = 882 \text{ lbf} \]

\[ F_{T,\text{water}} = W - F_B = 6984 - 882 = 6102 \text{ lbf} \approx 6100 \text{ lbf} \]

Discussion

Note that the weight of the concrete block and thus the tension of the rope decreases by \((6984 - 6102)/6984 = 12.6\%\) in water.

Section 3-97

Solution

An irregularly shaped body is weighed in air and then in water with a spring scale. The volume and the average density of the body are to be determined.

Properties

We take the density of water to be 1000 kg/m³.

Assumptions

1. The buoyancy force in air is negligible.
2. The body is completely submerged in water.

Analysis

The mass of the body is

\[ m = \frac{W_{\text{air}}}{g} = \frac{7200 \text{ N}}{9.81 \text{ m/s}^2} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) = 733.9 \text{ kg} \]

The difference between the weights in air and in water is due to the buoyancy force in water,

\[ F_B = W_{\text{air}} - W_{\text{water}} = 7200 - 4790 = 2410 \text{ N} \]

Noting that \( F_B = \rho_{\text{water}} g V \), the volume of the body is determined to be

\[ V = \frac{F_B}{\rho_{\text{water}} g} = \frac{2410 \text{ N}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = 0.2457 \text{ m}^3 \approx 0.246 \text{ m}^3 \]

Then the density of the body becomes

\[ \rho = \frac{m}{V} = \frac{733.9 \text{ kg}}{0.2457 \text{ m}^3} = 2987 \text{ kg/m}^3 \approx 2990 \text{ kg/m}^3 \]

Discussion

The volume of the body can also be measured by observing the change in the volume of the container when the body is dropped in it (assuming the body is not porous).
3-98

Solution  The height of the portion of a cubic ice block that extends above the water surface is measured. The height of the ice block below the surface is to be determined.

Assumptions  1 The buoyancy force in air is negligible. 2 The top surface of the ice block is parallel to the surface of the sea.

Properties  The specific gravities of ice and seawater are given to be 0.92 and 1.025, respectively, and thus the corresponding densities are 920 kg/m$^3$ and 1025 kg/m$^3$.

Analysis  The weight of a body floating in a fluid is equal to the buoyant force acting on it (a consequence of vertical force balance from static equilibrium). Therefore, in this case the average density of the body must be equal to the density of the fluid since

$$W = F_B \quad \Rightarrow \quad \rho_{\text{body}} g V_{\text{total}} = \rho_{\text{fluid}} g V_{\text{submerged}}$$

The cross-sectional of a cube is constant, and thus the “volume ratio” can be replaced by “height ratio”. Then,

$$\frac{h_{\text{submerged}}}{h_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{fluid}}} \quad \Rightarrow \quad \frac{h}{h + 0.15} = \frac{\rho_{\text{ice}}}{\rho_{\text{water}}} \quad \Rightarrow \quad \frac{h}{h + 0.15} = \frac{0.92}{1.025}$$

where $h$ is the height of the ice block below the surface. Solving for $h$ gives

$$h = \frac{(0.92)(0.15)}{1.025 - 0.92} = 1.3143 \text{ m}$$

Thus, to three significant digits, $h = 131 \text{ cm}$.

Discussion  Note that 0.92/1.025 = 0.89756, so approximately 90% of the volume of an ice block remains under water. For symmetrical ice blocks this also represents the fraction of height that remains under water.
### Solution 3-99

**Problem Statement:** If only 10% of an iceberg is above the water surface, the density of the iceberg is to be determined.

**Assumptions:**
1. The buoyancy force in air is negligible.
2. Density changes in the water are negligible with depth.

**Properties:**
- The specific gravity of seawater is 1025 kg/m$^3$.

**Analysis:**
- The weight of a body floating in a fluid is equal to the buoyant force acting on it (a consequence of vertical force balance from static equilibrium). Therefore, in this case the average density of the body must be equal to the density of the seawater since
  \[ W = F_B \rightarrow \rho_{\text{iceberg}} g V_{\text{total}} = \rho_{\text{seawater}} g V_{\text{submerged}} \]
- In the present problem, the submerged volume is 90% of the total volume. Thus,
  \[ \rho_{\text{iceberg}} = \rho_{\text{seawater}} \frac{V_{\text{submerged}}}{V_{\text{total}}} = \left(1025 \frac{\text{kg}}{\text{m}^3}\right)(0.90) = 922.5 \frac{\text{kg}}{\text{m}^3} \]

**Discussion:**
- Since the densities of the iceberg and the seawater are so close, the vast majority of the iceberg is submerged, hence the age-old expression “just the tip of the iceberg”.

### Solution 3-100

**Problem Statement:** A man dives into a lake and tries to lift a large rock. The force that the man needs to apply to lift it from the bottom of the lake is to be determined.

**Assumptions:**
1. The rock is completely submerged in water.
2. The buoyancy force in air is negligible.

**Properties:**
- The density of granite rock is given to be 2700 kg/m$^3$. We take the density of water to be 1000 kg/m$^3$.

**Analysis:**
- The weight and volume of the rock are
  \[ W = mg = (170 \text{ kg})(9.81 \text{ m/s}^2) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 1668 \text{ N} \]
  \[ V = \frac{m}{\rho} = \frac{170 \text{ kg}}{2700 \text{ kg/m}^3} = 0.06296 \text{ m}^3 \]
- The buoyancy force acting on the rock is
  \[ F_B = \rho_{\text{water}} g V = \left(1000 \frac{\text{kg}}{\text{m}^3}\right)(9.81 \text{ m/s}^2)(0.06296 \text{ m}^3) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 618 \text{ N} \]
- The weight of a body submerged in water is equal to the weight of the body in air minus the buoyancy force,
  \[ W_{\text{in water}} = W_{\text{in air}} - F_B = 1668 - 618 = 1050 \text{ N} \]
- This force corresponds to a mass of
  \[ m = \frac{W_{\text{in water}}}{g} = \frac{1050 \text{ N}}{9.81 \text{ m/s}^2} \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 107 \text{ kg} \]. Therefore, a person who can lift 107 kg on earth can lift this rock in water.
Solution

An irregularly shaped crown is weighed in air and then in water with a spring scale. It is to be determined if the crown is made of pure gold.

Assumptions

1. The buoyancy force in air is negligible.
2. The crown is completely submerged in water.

Properties

We take the density of water to be 1000 kg/m$^3$. The density of gold is given to be 19300 kg/m$^3$.

Analysis

The mass of the crown is

$$m = \frac{W_{\text{air}}}{g} = \frac{31.4 \text{ N}}{9.81 \text{ m/s}^2} = 3.20 \text{ kg}$$

The difference between the weights in air and in water is due to the buoyancy force in water, and thus

$$F_B = W_{\text{air}} - W_{\text{water}} = 31.4 - 28.9 = 2.50 \text{ N}$$

Noting that $F_B = \rho_{\text{water}} g V$, the volume of the crown is determined to be

$$V = \frac{F_B}{\rho_{\text{water}} g} = \frac{2.50 \text{ N}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = 2.548 \times 10^{-4} \text{ m}^3$$

Then the density of the crown becomes

$$\rho = \frac{m}{V} = \frac{3.20 \text{ kg}}{2.548 \times 10^{-4} \text{ m}^3} = 12,560 \text{ kg/m}^3$$

which is considerably less than the density of gold. Therefore, the crown is NOT made of pure gold.

Discussion

This problem can also be solved without doing any under-water weighing as follows: We would weigh a bucket half-filled with water, and drop the crown into it. After marking the new water level, we would take the crown out, and add water to the bucket until the water level rises to the mark. We would weigh the bucket again. Dividing the weight difference by the density of water and $g$ will give the volume of the crown. Knowing both the weight and the volume of the crown, the density can easily be determined.
Solution  The average density of a person is determined by weighing the person in air and then in water. A relation is to be obtained for the volume fraction of body fat in terms of densities.

Assumptions  1 The buoyancy force in air is negligible. 2 The body is considered to consist of fat and muscle only. 3 The body is completely submerged in water, and the air volume in the lungs is negligible.

Analysis  The difference between the weights of the person in air and in water is due to the buoyancy force in water. Therefore,

\[ F_B = W_{\text{air}} - W_{\text{water}} \rightarrow \rho_{\text{water}} g V = W_{\text{air}} - W_{\text{water}} \]

Knowing the weights and the density of water, the relation above gives the volume of the person. Then the average density of the person can be determined from

\[ \rho_{\text{ave}} = \frac{m}{V} = \frac{W_{\text{air}}}{g V} \]

Under assumption #2, the total mass of a person is equal to the sum of the masses of the fat and muscle tissues, and the total volume of a person is equal to the sum of the volumes of the fat and muscle tissues. The volume fraction of body fat is the ratio of the fat volume to the total volume of the person. Therefore,

\[ V = V_{\text{fat}} + V_{\text{muscle}} \quad \text{where} \quad V_{\text{fat}} = x_{\text{fat}} V \quad \text{and} \quad V_{\text{muscle}} = x_{\text{muscle}} V = (1 - x_{\text{fat}})V \]

\[ m = m_{\text{fat}} + m_{\text{muscle}} \]

Noting that mass is density times volume, the last relation can be written as

\[ \rho_{\text{ave}} V = \rho_{\text{fat}} V_{\text{fat}} + \rho_{\text{muscle}} V_{\text{muscle}} \]

\[ \rho_{\text{ave}} V = \rho_{\text{fat}} x_{\text{fat}} V + \rho_{\text{muscle}} (1 - x_{\text{fat}}) V \]

Canceling the \( V \) and solving for \( x_{\text{fat}} \) gives the desired relation,

\[ x_{\text{fat}} = \frac{\rho_{\text{muscle}} - \rho_{\text{ave}}}{\rho_{\text{muscle}} - \rho_{\text{fat}}} \]

Discussion  Weighing a person in water in order to determine its volume is not practical. A more practical way is to use a large container, and measuring the change in volume when the person is completely submerged in it.
Solution  
The volume of the hull of a boat is given. The amounts of load the boat can carry in a lake and in the sea are to be determined.

Assumptions  
1. The dynamic effects of the waves are disregarded.  
2. The buoyancy force in air is negligible.

Properties  
The density of sea water is given to be $1.03 \times 1000 = 1030\,\text{kg/m}^3$. We take the density of water to be $1000\,\text{kg/m}^3$.

Analysis  
The weight of the unloaded boat is

$$W_{\text{boat}} = mg = (8560\,\text{kg})(9.81\,\text{m/s}^2) = 84.0\,\text{kN}$$

The buoyancy force becomes a maximum when the entire hull of the boat is submerged in water, and is determined to be

$$F_B,\text{lake} = \rho_{\text{lake}} g V = (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(180\,\text{m}^3) = 1766\,\text{kN}$$

$$F_B,\text{sea} = \rho_{\text{sea}} g V = (1030\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(180\,\text{m}^3) = 1819\,\text{kN}$$

The total weight of a floating boat (load + boat itself) is equal to the buoyancy force. Therefore, the weight of the maximum load is

$$W_{\text{load, lake}} = F_B,\text{lake} - W_{\text{boat}} = 1766 - 84 = 1682\,\text{kN}$$

$$W_{\text{load, sea}} = F_B,\text{sea} - W_{\text{boat}} = 1819 - 84 = 1735\,\text{kN}$$

The corresponding masses of load are

$$m_{\text{load, lake}} = \frac{W_{\text{load, lake}}}{g} = \frac{1682\,\text{kN}}{9.81\,\text{m/s}^2} = 171,500\,\text{kg}$$

$$m_{\text{load, sea}} = \frac{W_{\text{load, sea}}}{g} = \frac{1735\,\text{kN}}{9.81\,\text{m/s}^2} = 176,900\,\text{kg}$$

Discussion  
Note that this boat can carry nearly 5400 kg more load in the sea than it can in fresh water. Fully-loaded boats in sea water should expect to sink into water deeper when they enter fresh water, such as a river where the port may be.
**Fluids in Rigid-Body Motion**

3-104C
Solution  We are to discuss when a fluid can be treated as a rigid body.

*Analysis*  A moving body of fluid can be treated as a rigid body when there are no shear stresses (i.e., no motion between fluid layers relative to each other) in the fluid body.

*Discussion*  When there is no relative motion between fluid particles, there are no viscous stresses, and pressure (normal stress) is the only stress.

3-105C
Solution  We are to compare the pressure at the bottom of a glass of water moving at various velocities.

*Analysis*  The water pressure at the bottom surface is the same for all cases since the acceleration for all four cases is zero.

*Discussion*  When any body, fluid or solid, moves at constant velocity, there is no acceleration, regardless of the direction of the movement.

3-106C
Solution  We are to compare the pressure in a glass of water for stationary and accelerating conditions.

*Analysis*  The pressure at the bottom surface is constant when the glass is stationary. For a glass moving on a horizontal plane with constant acceleration, water will collect at the back but the water depth will remain constant at the center. Therefore, the pressure at the midpoint will be the same for both glasses. But the bottom pressure will be low at the front relative to the stationary glass, and high at the back (again relative to the stationary glass). Note that the pressure in all cases is the hydrostatic pressure, which is directly proportional to the fluid height.

*Discussion*  We ignore any sloshing of the water.

3-107C
Solution  We are to analyze the pressure in a glass of water that is rotating.

*Analysis*  When a vertical cylindrical container partially filled with water is rotated about its axis and rigid body motion is established, the fluid level will drop at the center and rise towards the edges. Noting that hydrostatic pressure is proportional to fluid depth, the pressure at the midpoint will drop and the pressure at the edges of the bottom surface will rise due to the rotation.

*Discussion*  The highest pressure occurs at the bottom corners of the container.
Solution
A water tank is being towed by a truck on a level road, and the angle the free surface makes with the horizontal is measured. The acceleration of the truck is to be determined.

Assumptions
1. The road is horizontal so that acceleration has no vertical component \( (a_z = 0) \).
2. Effects of splashing, breaking, driving over bumps, and climbing hills are assumed to be secondary, and are not considered.
3. The acceleration remains constant.

Analysis
We take the \( x \)-axis to be the direction of motion, the \( z \)-axis to be the upward vertical direction. The tangent of the angle the free surface makes with the horizontal is

\[
\tan \theta = \frac{dx}{dz}
\]

Solving for \( a_x \) and substituting,

\[
a_x = (g + a_z) \tan \theta = (9.81 \text{ m/s}^2 + 0) \tan 12^\circ = 2.09 \text{ m/s}^2
\]

Discussion
Note that the analysis is valid for any fluid with constant density since we used no information that pertains to fluid properties in the solution.

Solution
Two water tanks filled with water, one stationary and the other moving upwards at constant acceleration. The tank with the higher pressure at the bottom is to be determined.

Assumptions
1. The acceleration remains constant.
2. Water is an incompressible substance.

Properties
We take the density of water to be 1000 kg/m³.

Analysis
The pressure difference between two points 1 and 2 in an incompressible fluid is given by

\[
P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho (g + a_z) (z_2 - z_1)
\]

since \( a_z = 0 \). Taking point 2 at the free surface and point 1 at the tank bottom, we have \( P_2 = P_{\text{atm}} \) and \( z_2 - z_1 = h \) and thus

\[
P_{1, \text{gage}} = P_{\text{bottom}} = \rho (g + a_z) h
\]

Tank A: We have \( a_z = 0 \), and thus the pressure at the bottom is

\[
P_{A, \text{bottom}} = \rho gh_A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(8 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right) = 78.5 \text{ kN/m}^2
\]

Tank B: We have \( a_z = +5 \text{ m/s}^2 \), and thus the pressure at the bottom is

\[
P_{B, \text{bottom}} = \rho (g + a_z) h_B = (1000 \text{ kg/m}^3)(9.81 + 5 \text{ m/s}^2)(2 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right) = 29.6 \text{ kN/m}^2
\]

Therefore, tank A has a higher pressure at the bottom.

Discussion
We can also solve this problem quickly by examining the relation \( P_{\text{bottom}} = \rho (g + a_z) h \). Acceleration for tank B is about 1.5 times that of Tank A (14.81 vs 9.81 m/s²), but the fluid depth for tank A is 4 times that of tank B (8 m vs 2 m). Therefore, the tank with the larger acceleration-fluid height product (tank A in this case) will have a higher pressure at the bottom.
**Solution**  A water tank is being towed on an uphill road at constant acceleration. The angle the free surface of water makes with the horizontal is to be determined, and the solution is to be repeated for the downhill motion case.

**Assumptions**  1 Effects of splashing, breaking, driving over bumps, and climbing hills are assumed to be secondary, and are not considered.  2 The acceleration remains constant.

**Analysis**  We take the $x$- and $z$-axes as shown in the figure. From geometrical considerations, the horizontal and vertical components of acceleration are

\[
\begin{align*}
a_x &= a \cos \alpha \\
a_z &= a \sin \alpha
\end{align*}
\]

The tangent of the angle the free surface makes with the horizontal is

\[
\tan \theta = \frac{a_x}{g + a_z} = \frac{a \cos \alpha}{g + a \sin \alpha} = \frac{(5 \text{ m/s}^2) \cos 20^\circ}{9.81 \text{ m/s}^2 + (5 \text{ m/s}^2) \sin 20^\circ} = 0.4078 \quad \rightarrow \quad \theta = 22.2^\circ
\]

When the direction of motion is reversed, both $a_x$ and $a_z$ are in negative $x$- and $z$-direction, respectively, and thus become negative quantities,

\[
\begin{align*}
a_x &= -a \cos \alpha \\
a_z &= -a \sin \alpha
\end{align*}
\]

Then the tangent of the angle the free surface makes with the horizontal becomes

\[
\tan \theta = \frac{a_x}{g + a_z} = \frac{-a \cos \alpha}{g + a \sin \alpha} = \frac{-(5 \text{ m/s}^2) \cos 20^\circ}{9.81 \text{ m/s}^2 - (5 \text{ m/s}^2) \sin 20^\circ} = -0.5801 \quad \rightarrow \quad \theta = -30.1^\circ
\]

**Discussion**  Note that the analysis is valid for any fluid with constant density, not just water, since we used no information that pertains to water in the solution.
3-111E

Solution  A vertical cylindrical tank open to the atmosphere is rotated about the centerline. The angular velocity at which the bottom of the tank will first be exposed, and the maximum water height at this moment are to be determined.

Assumptions  1 The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body.  2 Water is an incompressible fluid.

Analysis  Taking the center of the bottom surface of the rotating vertical cylinder as the origin \((r = 0, z = 0)\), the equation for the free surface of the liquid is given as

\[
z_s(r) = h_0 - \frac{\omega^2}{4g} (R^2 - 2r^2)
\]

where \(h_0 = 1\) ft is the original height of the liquid before rotation. Just before dry spot appear at the center of bottom surface, the height of the liquid at the center equals zero, and thus \(z_s(0) = 0\). Solving the equation above for \(\omega\) and substituting,

\[
\omega = \sqrt{\frac{2gR}{R^2 - \frac{2}{2}R^2}} = \sqrt{\frac{4(32.2 \text{ ft/s}^2)(1 \text{ ft})}{(1.5 \text{ ft})^2}} = 7.566 \text{ rad/s} \approx 7.57 \text{ rad/s}
\]

Noting that one complete revolution corresponds to \(2\pi\) radians, the rotational speed of the container can also be expressed in terms of revolutions per minute (rpm) as

\[
\dot{n} = \frac{\omega}{2\pi} = \frac{7.566 \text{ rad/s}}{2\pi \text{ rad/rev}} \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 72.3 \text{ rpm}
\]

Therefore, the rotational speed of this container should be limited to 72.3 rpm to avoid any dry spots at the bottom surface of the tank.

The maximum vertical height of the liquid occurs a the edges of the tank \((r = R = 1 \text{ ft})\), and it is

\[
z_s(R) = h_0 + \frac{\omega^2 R^2}{4g} = (1 \text{ ft}) + \frac{(7.566 \text{ rad/s})^2(1.5 \text{ ft})^2}{4(32.2 \text{ ft/s}^2)} = 2.00 \text{ ft}
\]

Discussion  Note that the analysis is valid for any liquid since the result is independent of density or any other fluid property.
**Solution**  A cylindrical tank is being transported on a level road at constant acceleration. The allowable water height to avoid spill of water during acceleration is to be determined.

\[ a_x = 4 \text{ m/s}^2 \]

**Assumptions**  1 The road is horizontal during acceleration so that acceleration has no vertical component \((a_z = 0)\). 2 Effects of splashing, breaking, driving over bumps, and climbing hills are assumed to be secondary, and are not considered. 3 The acceleration remains constant.

**Analysis**  We take the \(x\)-axis to be the direction of motion, the \(z\)-axis to be the upward vertical direction, and the origin to be the midpoint of the tank bottom. The tangent of the angle the free surface makes with the horizontal is

\[ \tan \theta = \frac{a_x}{g + a_z} = \frac{4}{9.81 + 0} = 0.4077 \quad \text{(and thus } \theta = 22.2^\circ) \]

The maximum vertical rise of the free surface occurs at the back of the tank, and the vertical midplane experiences no rise or drop during acceleration. Then the maximum vertical rise at the back of the tank relative to the midplane is

\[ \Delta z_{\text{max}} = (D/2) \tan \theta = ([0.40 \text{ m}]/2) \times 0.4077 = 0.082 \text{ m} = 8.2 \text{ cm} \]

Therefore, the maximum initial water height in the tank to avoid spilling is

\[ h_{\text{max}} = h_{\text{tank}} - \Delta z_{\text{max}} = 60 - 8.2 = 51.8 \text{ cm} \]

**Discussion**  Note that the analysis is valid for any fluid with constant density, not just water, since we used no information that pertains to water in the solution.
Solution  A vertical cylindrical container partially filled with a liquid is rotated at constant speed. The drop in the liquid level at the center of the cylinder is to be determined.

Assumptions  1 The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. 2 The bottom surface of the container remains covered with liquid during rotation (no dry spots).

Analysis  Taking the center of the bottom surface of the rotating vertical cylinder as the origin \((r = 0, z = 0)\), the equation for the free surface of the liquid is given as

\[
zs(r) = h_0 - \frac{\omega^2}{4g} (R^2 - 2r^2)
\]

where \(h_0 = 0.6 \text{ m}\) is the original height of the liquid before rotation, and

\[
\omega = 2\pi \dot{\theta} = 2\pi (120 \text{ rev/min}) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 12.57 \text{ rad/s}
\]

Then the vertical height of the liquid at the center of the container where \(r = 0\) becomes

\[
zs(0) = h_0 - \frac{\omega^2 R^2}{4g} = (0.60 \text{ m}) - \frac{(12.57 \text{ rad/s})^2 (0.20 \text{ m})^2}{4(9.81 \text{ m/s}^2)} = 0.44 \text{ m}
\]

Therefore, the drop in the liquid level at the center of the cylinder is

\[
\Delta h_{\text{drop, center}} = h_0 - zs(0) = 0.60 - 0.44 = 0.16 \text{ m}
\]

Discussion  Note that the analysis is valid for any liquid since the result is independent of density or any other fluid property. Also, our assumption of no dry spots is validated since \(z_0(0)\) is positive.
The motion of a fish tank in the cabin of an elevator is considered. The pressure at the bottom of the tank when the elevator is stationary, moving up with a specified acceleration, and moving down with a specified acceleration is to be determined.

**Assumptions**
1. The acceleration remains constant.
2. Water is an incompressible substance.

**Properties**
We take the density of water to be 1000 kg/m³.

**Analysis**
The pressure difference between two points 1 and 2 in an incompressible fluid is given by

\[ P_2 - P_1 = -\rho a_z (x_2 - x_1) - \rho (g + a_z)(z_2 - z_1) \]

or

\[ P_1 - P_2 = \rho (g + a_z)(z_2 - z_1) \]

since \( a_z = 0 \). Taking point 2 at the free surface and point 1 at the tank bottom, we have \( P_2 = P_{atm} \) and \( z_2 - z_1 = h \) and thus

\[ P_{1,\text{gage}} = P_{\text{bottom}} = \rho (g + a_z)h \]

(a) **Tank stationary:** We have \( a_z = 0 \), and thus the gage pressure at the tank bottom is

\[ P_{\text{bottom}} = \rho gh = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.6 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 5.89 \text{ kN/m}^2 = 5.89 \text{ kPa} \]

(b) **Tank moving up:** We have \( a_z = +3 \text{ m/s}^2 \), and thus the gage pressure at the tank bottom is

\[ P_{\text{bottom}} = \rho (g + a_z)h_B = (1000 \text{ kg/m}^3)(9.81 + 3 \text{ m/s}^2)(0.6 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 7.69 \text{ kN/m}^2 = 7.69 \text{ kPa} \]

(c) **Tank moving down:** We have \( a_z = -3 \text{ m/s}^2 \), and thus the gage pressure at the tank bottom is

\[ P_{\text{bottom}} = \rho (g + a_z)h_B = (1000 \text{ kg/m}^3)(9.81 - 3 \text{ m/s}^2)(0.6 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 4.09 \text{ kN/m}^2 = 4.09 \text{ kPa} \]

**Discussion**
Note that the pressure at the tank bottom while moving up in an elevator is almost twice that while moving down, and thus the tank is under much greater stress during upward acceleration.
Solution  A vertical cylindrical milk tank is rotated at constant speed, and the pressure at the center of the bottom surface is measured. The pressure at the edge of the bottom surface is to be determined.

Assumptions  1. The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. 2. Milk is an incompressible substance.

Properties  The density of the milk is given to be $1030 \text{ kg/m}^3$.

Analysis  Taking the center of the bottom surface of the rotating vertical cylinder as the origin ($r = 0, z = 0$), the equation for the free surface of the liquid is given as

$$z_s(r) = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

where $R = 1.5 \text{ m}$ is the radius, and

$$\omega = 2\pi n = 2\pi(12 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 1.2566 \text{ rad/s}$$

The fluid rise at the edge relative to the center of the tank is

$$\Delta h = z_s(R) - z_s(0) = \left(h_0 - \frac{\omega^2 R^2}{4g}\right) - \left(h_0 - \frac{\omega^2 R^2}{4g}\right) = \frac{\omega^2 R^2}{2g} = \frac{(1.2566 \text{ rad/s})^2 (1.50 \text{ m})^2}{2(9.81 \text{ m/s}^2)} = 1.1811 \text{ m}$$

The pressure difference corresponding to this fluid height difference is

$$\Delta P_{\text{bottom}} = \rho g \Delta h = (1030 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.1811 \text{ m}) \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right) = 1.83 \text{ kN/m}^2 = 1.83 \text{ kPa}$$

Then the pressure at the edge of the bottom surface becomes

$$P_{\text{bottom, edge}} = P_{\text{bottom, center}} + \Delta P_{\text{bottom}} = 130 + 1.83 = 131.83 \text{ kPa} \approx 132 \text{ kPa}$$

Discussion  Note that the pressure is 1.4% higher at the edge relative to the center of the tank, and there is a fluid level difference of 1.18 m between the edge and center of the tank, and these differences should be considered when designing rotating fluid tanks.
Solution  Milk is transported in a completely filled horizontal cylindrical tank accelerating at a specified rate. The maximum pressure difference in the tanker is to be determined.

Assumptions  1 The acceleration remains constant. 2 Milk is an incompressible substance.

Properties  The density of the milk is given to be 1020 kg/m³.

Analysis  We take the x- and z- axes as shown. The horizontal acceleration is in the negative x direction, and thus a_x is negative. Also, there is no acceleration in the vertical direction, and thus a_z = 0. The pressure difference between two points 1 and 2 in an incompressible fluid in linear rigid body motion is given by

\[
P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho (g + a_z) (z_2 - z_1) \quad \rightarrow \quad P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho g (z_2 - z_1)
\]

The first term is due to acceleration in the horizontal direction and the resulting compression effect towards the back of the tanker, while the second term is simply the hydrostatic pressure that increases with depth. Therefore, we reason that the lowest pressure in the tank will occur at point 1 (upper front corner), and the higher pressure at point 2 (the lower rear corner). Therefore, the maximum pressure difference in the tank is

\[
\Delta P_{\text{max}} = P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho g (z_2 - z_1) = -[a_x (x_2 - x_1) + g (z_2 - z_1)]
\]

\[
= -(1020 \text{ kg/m}^3)(-3 \text{ m/s}^2)(7 \text{ m}) + (9.81 \text{ m/s}^2)(-3 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)
\]

\[
= (17.9 + 30.0) \text{ kN/m}^2 = 47.9 \text{ kPa}
\]

since \(x_1 = 0\), \(x_2 = 7 \text{ m}\), \(z_1 = 3 \text{ m}\), and \(z_2 = 0\).

Discussion  Note that the variation of pressure along a horizontal line is due to acceleration in the horizontal direction while the variation of pressure in the vertical direction is due to the effects of gravity and acceleration in the vertical direction (which is zero in this case).
Chapter 3 Pressure and Fluid Statics

3-117

Solution  Milk is transported in a completely filled horizontal cylindrical tank decelerating at a specified rate. The maximum pressure difference in the tanker is to be determined.

Assumptions  1 The acceleration remains constant. 2 Milk is an incompressible substance.

Properties  The density of the milk is given to be 1020 kg/m³.

Analysis  We take the x- and z- axes as shown. The horizontal deceleration is in the x direction, and thus \( a_x \) is positive. Also, there is no acceleration in the vertical direction, and thus \( a_z = 0 \). The pressure difference between two points 1 and 2 in an incompressible fluid in linear rigid body motion is given by

\[
P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \quad \Rightarrow \quad P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho g(z_2 - z_1)
\]

The first term is due to deceleration in the horizontal direction and the resulting compression effect towards the front of the tanker, while the second term is simply the hydrostatic pressure that increases with depth. Therefore, we reason that the lowest pressure in the tank will occur at point 1 (upper front corner), and the higher pressure at point 2 (the lower rear corner). Therefore, the maximum pressure difference in the tank is

\[
\Delta P_{\text{max}} = P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho g(z_2 - z_1) = -[a_x (x_2 - x_1) + g(z_2 - z_1)]
\]

\[
= -\left(1020 \text{ kg/m}^3 \right) \left(2.5 \text{ m/s}^2 \right) (-7 \text{ m}) + (9.81 \text{ m/s}^2) (-3 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right)
\]

\[
= (17.9 + 30.0) \text{ kN/m}^2 = 47.9 \text{ kPa}
\]

Discussion  Note that the variation of pressure along a horizontal line is due to acceleration in the horizontal direction while the variation of pressure in the vertical direction is due to the effects of gravity and acceleration in the vertical direction (which is zero in this case).

3-118

Solution  A vertical U-tube partially filled with alcohol is rotated at a specified rate about one of its arms. The elevation difference between the fluid levels in the two arms is to be determined.

Assumptions  1 Alcohol is an incompressible fluid.

Analysis  Taking the base of the left arm of the U-tube as the origin \((r = 0, z = 0)\), the equation for the free surface of the liquid is given as

\[
z_s(r) = h_0 - \frac{\omega^2 R^2}{4g} (R^2 - 2r^2)
\]

where \( h_0 = 0.20 \text{ m} \) is the original height of the liquid before rotation, and \( \omega = 4.2 \text{ rad/s} \). The fluid rise at the right arm relative to the fluid level in the left arm (the center of rotation) is

\[
\Delta h = z_s(R) - z_s(0) = \left( h_0 + \frac{\omega^2 R^2}{4g} \right) - \left( h_0 \frac{\omega^2 R^2}{4g} \right) = \frac{\omega^2 R^2}{2g} = \frac{(4.2 \text{ rad/s})^2 (0.30 \text{ m})^2}{2(9.81 \text{ m/s}^2)} = 0.081 \text{ m}
\]

Discussion  The analysis is valid for any liquid since the result is independent of density or any other fluid property.
**Chapter 3 Pressure and Fluid Statics**

**Solution**  A vertical cylindrical tank is completely filled with gasoline, and the tank is rotated about its vertical axis at a specified rate. The pressures difference between the centers of the bottom and top surfaces, and the pressures difference between the center and the edge of the bottom surface are to be determined.

![Diagram of a cylindrical tank](image)

**Assumptions**  1. The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body.
   2. Gasoline is an incompressible substance.

**Properties**  The density of the gasoline is given to be 740 kg/m³.

**Analysis**  The pressure difference between two points 1 and 2 in an incompressible fluid rotating in rigid body motion is given by

\[ P_2 - P_1 = \rho \omega^2 \left( r_2^2 - r_1^2 \right) - \rho g (z_2 - z_1) \]

where \( R = 0.60 \) m is the radius, and

\[ \omega = 2\pi \frac{n}{60} \left( \frac{1 \text{ min}}{1 \text{ rev/min}} \right) = 7.330 \text{ rad/s} \]

(a) Taking points 1 and 2 to be the centers of the bottom and top surfaces, respectively, we have \( r_1 = r_2 = 0 \) and \( z_2 - z_1 = h = 3 \) m. Then,

\[ P_{\text{center, top}} - P_{\text{center, bottom}} = 0 - \rho g (z_2 - z_1) = -\rho g h \]

\[ = -(740 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 21.8 \text{ kN/m}^2 = 21.8 \text{ kPa} \]

(b) Taking points 1 and 2 to be the center and edge of the bottom surface, respectively, we have \( r_1 = 0 \), \( r_2 = R \), and \( z_2 = z_1 = 0 \). Then,

\[ P_{\text{edge, bottom}} - P_{\text{center, bottom}} = \frac{\rho \omega^2}{2} (R_2^2 - 0) - 0 = \frac{\rho \omega^2 R^2}{2} \]

\[ = (740 \text{ kg/m}^3)(7.33 \text{ rad/s})^2 (0.60 \text{ m})^2 \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 7.16 \text{ kN/m}^2 = 7.16 \text{ kPa} \]

**Discussion**  Note that the rotation of the tank does not affect the pressure difference along the axis of the tank. But the pressure difference between the edge and the center of the bottom surface (or any other horizontal plane) is due entirely to the rotation of the tank.
Solution  The previous problem is reconsidered. The effect of rotational speed on the pressure difference between the center and the edge of the bottom surface of the cylinder as the rotational speed varies from 0 to 500 rpm in increments of 50 rpm is to be investigated.

Analysis  The EES Equations window is printed below, followed by the tabulated and plotted results.

\[ g = 9.81 \text{ m/s}^2 \]
\[ \rho = 740 \text{ kg/m}^3 \]
\[ R = 0.6 \text{ m} \]
\[ h = 3 \text{ m} \]

\[ \omega = 2\pi n_{dot}/60 \text{ rad/s} \]
\[ \Delta P_{\text{center-edge}} = \rho g h/1000 \text{ kPa} \]
\[ \Delta P_{\text{bottom}} = \rho \omega^2 R^2/2000 \text{ kPa} \]

<table>
<thead>
<tr>
<th>Rotation rate ( n ), rpm</th>
<th>Angular speed ( \omega ), rad/s</th>
<th>( \Delta P_{\text{center-edge}} ), kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>0.0</td>
</tr>
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<td>50</td>
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<tr>
<td>400</td>
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<td>233.7</td>
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<td>295.8</td>
</tr>
<tr>
<td>500</td>
<td>52.4</td>
<td>365.2</td>
</tr>
</tbody>
</table>

Discussion  The pressure rise with rotation rate is not linear, but rather quadratic.
3-121E

**Solution** A water tank partially filled with water is being towed by a truck on a level road. The maximum acceleration (or deceleration) of the truck to avoid spilling is to be determined.

**Assumptions** 1 The road is horizontal so that acceleration has no vertical component \( (a_z = 0) \). 2 Effects of splashing, breaking, driving over bumps, and climbing hills are assumed to be secondary, and are not considered. 3 The acceleration remains constant.

**Analysis** We take the \( x \)-axis to be the direction of motion, the \( z \)-axis to be the upward vertical direction. The shape of the free surface just before spilling is shown in figure. The tangent of the angle the free surface makes with the horizontal is given by

\[
\tan \theta = \frac{a_x}{g + a_z} \quad \Rightarrow \quad a_x = g \tan \theta
\]

where \( a_z = 0 \) and, from geometric considerations, \( \tan \theta = \frac{\Delta h}{L/2} \). Substituting, we get

\[
a_x = g \tan \theta = g \frac{\Delta h}{L/2} = \frac{(32.2 \text{ ft/s}^2)}{(22 \text{ ft})/2} = 2.93 \text{ ft/s}^2
\]

The solution can be repeated for deceleration by replacing \( a_x \) by \(-a_x\). We obtain \( a_x = -2.93 \text{ m/s}^2 \).

**Discussion** Note that the analysis is valid for any fluid with constant density since we used no information that pertains to fluid properties in the solution.

---

3-122E

**Solution** A water tank partially filled with water is being towed by a truck on a level road. The maximum acceleration (or deceleration) of the truck to avoid spilling is to be determined.

**Assumptions** 1 The road is horizontal so that deceleration has no vertical component \( (a_z = 0) \). 2 Effects of splashing and driving over bumps are assumed to be secondary, and are not considered. 3 The deceleration remains constant.

**Analysis** We take the \( x \)-axis to be the direction of motion, the \( z \)-axis to be the upward vertical direction. The shape of the free surface just before spilling is shown in figure. The tangent of the angle the free surface makes with the horizontal is given by

\[
\tan \theta = -\frac{a_x}{g + a_z} \quad \Rightarrow \quad a_x = -g \tan \theta
\]

where \( a_z = 0 \) and, from geometric considerations, \( \tan \theta = \frac{\Delta h}{L/2} \). Substituting, we get

\[
a_x = -g \tan \theta = -g \frac{\Delta h}{L/2} = -(32.2 \text{ ft/s}^2) \frac{0.5 \text{ ft}}{(8 \text{ ft})/2} = -4.025 \text{ ft/s}^2 \approx -4.03 \text{ ft/s}^2
\]

**Discussion** Note that the analysis is valid for any fluid with constant density since we used no information that pertains to fluid properties in the solution.
Solution  Water is transported in a completely filled horizontal cylindrical tanker accelerating at a specified rate. The pressure difference between the front and back ends of the tank along a horizontal line when the truck accelerates and decelerates at specified rates.

Assumptions  1 The acceleration remains constant. 2 Water is an incompressible substance.

Properties  We take the density of the water to be 1000 kg/m$^3$.

Analysis  (a) We take the $x$- and $z$- axes as shown. The horizontal acceleration is in the negative $x$ direction, and thus $a_x$ is negative. Also, there is no acceleration in the vertical direction, and thus $a_z = 0$. The pressure difference between two points 1 and 2 in an incompressible fluid in linear rigid body motion is given by

$$ P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho (g + a_z)(z_2 - z_1) \quad \rightarrow \quad P_2 - P_1 = -\rho a_x (x_2 - x_1) $$

since $z_2 - z_1 = 0$ along a horizontal line. Therefore, the pressure difference between the front and back of the tank is due to acceleration in the horizontal direction and the resulting compression effect towards the back of the tank. Then the pressure difference along a horizontal line becomes

$$ \Delta P = P_2 - P_1 = -\rho a_x (x_2 - x_1) = -(1000 \text{ kg/m}^3)(-3 \text{ m/s}^2)(7 \text{ m}) \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right) = 21 \text{ kN/m}^2 = 21 \text{ kPa} $$

since $x_1 = 0$ and $x_2 = 7 \text{ m}$.

(b) The pressure difference during deceleration is determined the way, but $a_x = 4 \text{ m/s}^2$ in this case,

$$ \Delta P = P_2 - P_1 = -\rho a_x (x_2 - x_1) = -(1000 \text{ kg/m}^3)(4 \text{ m/s}^2)(7 \text{ m}) \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right) = -28 \text{ kN/m}^2 = -28 \text{ kPa} $$

Discussion  Note that the pressure is higher at the back end of the tank during acceleration, but at the front end during deceleration (during breaking, for example) as expected.
Review Problems

3-124
Solution One section of the duct of an air-conditioning system is laid underwater. The upward force the water exerts on the duct is to be determined.

Assumptions 1 The diameter given is the outer diameter of the duct (or, the thickness of the duct material is negligible). 2 The weight of the duct and the air in is negligible.

Properties The density of air is given to be \( \rho = 1.30 \text{ kg/m}^3 \). We take the density of water to be 1000 kg/m\(^3\).

Analysis Noting that the weight of the duct and the air in it is negligible, the net upward force acting on the duct is the buoyancy force exerted by water. The volume of the underground section of the duct is

\[
V = AL = (\pi D^2 / 4) L = [\pi (0.15 \text{ m})^2 / 4] (20 \text{ m}) = 0.3534 \text{ m}^3
\]

Then the buoyancy force becomes

\[
F_B = \rho g V = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3534 \text{ m}^3) = 3.47 \text{ kN}
\]

Discussion The upward force exerted by water on the duct is 3.47 kN, which is equivalent to the weight of a mass of 354 kg. Therefore, this force must be treated seriously.

3-125
Solution A helium balloon tied to the ground carries 2 people. The acceleration of the balloon when it is first released is to be determined.

Assumptions The weight of the cage and the ropes of the balloon is negligible.

Properties The density of air is given to be \( \rho = 1.16 \text{ kg/m}^3 \). The density of helium gas is 1/7th of this.

Analysis The buoyancy force acting on the balloon is

\[
V_{\text{balloon}} = 4\pi r^3 / 3 = 4\pi (6 \text{ m})^3 / 3 = 904.8 \text{ m}^3
\]

\[
F_B = \rho_{\text{air}} g V_{\text{balloon}}
\]

\[
= (1.16 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(904.8 \text{ m}^3)\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 10,296 \text{ N}
\]

The total mass is

\[
m_{\text{He}} = \rho_{\text{He}} V = \left(\frac{1.16}{7}\right) \text{kg/m}^3(904.8 \text{ m}^3) = 149.9 \text{ kg}
\]

\[
m_{\text{total}} = m_{\text{He}} + m_{\text{people}} = 149.9 + 2 \times 70 = 289.9 \text{ kg}
\]

The total weight is

\[
W = m_{\text{total}} g = (289.9 \text{ kg})(9.81 \text{ m/s}^2)\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 2844 \text{ N}
\]

Thus the net force acting on the balloon is

\[
F_{\text{net}} = F_B - W = 10,296 - 2844 = 7452 \text{ N}
\]

Then the acceleration becomes

\[
a = \frac{F_{\text{net}}}{m_{\text{total}}} = \frac{7452 \text{ N}}{289.9 \text{ kg}}\left(\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}}\right) = 25.7 \text{ m/s}^2
\]

Discussion This is almost three times the acceleration of gravity – aerodynamic drag on the balloon acts quickly to slow down the acceleration.
Solution  The previous problem is reconsidered. The effect of the number of people carried in the balloon on acceleration is to be investigated. Acceleration is to be plotted against the number of people, and the results are to be discussed.

Analysis  The EES Equations window is printed below, followed by the tabulated and plotted results.

"Given Data:
ρ_air=1.16\,[kg/m^3]  "density of air"
g=9.807\,[m/s^2]"
d_balloon=10\,[m]"
m_1person=70\,[kg]"
{NoPeople = 2}  "Data supplied in Parametric Table"

"Calculated values:
ρ_He=ρ_air/7\,[kg/m^3]  "density of helium"
r_balloon=d_balloon/2\,[m]"
V_balloon=4*π*r_balloon^3/3\,[m^3]"
m_people=NoPeople*m_1person\,[kg]"
m_He=ρ_He*V_balloon\,[kg]"
m_total=m_He+m_people\,[kg]"
"The total weight of balloon and people is:
W_total=m_total*g\,[N]"
"The buoyancy force acting on the balloon, F_b, is equal to the weight of the air displaced by the balloon."
F_b=ρ_air*V_balloon*g\,[N]"
"From the free body diagram of the balloon, the balancing vertical forces must equal the product of the total mass and the vertical acceleration."
F_b- W_total=m_total*a_up

<table>
<thead>
<tr>
<th>A_up [m/s^2]</th>
<th>No. People</th>
</tr>
</thead>
<tbody>
<tr>
<td>28.19</td>
<td>1</td>
</tr>
<tr>
<td>16.46</td>
<td>2</td>
</tr>
<tr>
<td>10.26</td>
<td>3</td>
</tr>
<tr>
<td>6.434</td>
<td>4</td>
</tr>
<tr>
<td>3.831</td>
<td>5</td>
</tr>
<tr>
<td>1.947</td>
<td>6</td>
</tr>
<tr>
<td>0.5204</td>
<td>7</td>
</tr>
<tr>
<td>-0.5973</td>
<td>8</td>
</tr>
<tr>
<td>-1.497</td>
<td>9</td>
</tr>
<tr>
<td>-2.236</td>
<td>10</td>
</tr>
</tbody>
</table>

Discussion  As expected, the more people, the slower the acceleration. In fact, if more than 7 people are on board, the balloon does not rise at all.
Solution

A balloon is filled with helium gas. The maximum amount of load the balloon can carry is to be determined.

Assumptions

The weight of the cage and the ropes of the balloon is negligible.

Properties

The density of air is given to be $\rho = 1.16 \text{ kg/m}^3$. The density of helium gas is $1/7$th of this.

Analysis

In the limiting case, the net force acting on the balloon will be zero. That is, the buoyancy force and the weight will balance each other:

$$W = mg = F_B$$

$$m_{\text{total}} = \frac{F_B}{g} = \frac{5958.4 \text{ N}}{9.81 \text{ m/s}^2} = 607.4 \text{ kg}$$

Thus,

$$m_{\text{people}} = m_{\text{total}} - m_{\text{He}} = 607.4 - 86.8 = 520.6 \text{ kg} \approx 521 \text{ kg}$$

Discussion

When the net weight of the balloon and its cargo exceeds the weight of the air it displaces, the balloon/cargo is no longer “lighter than air”, and therefore cannot rise.

---

Solution

The pressure in a steam boiler is given in kgf/cm$^2$. It is to be expressed in psi, kPa, atm, and bars.

Analysis

We note that 1 atm = 1.03323 kgf/cm$^2$, 1 atm = 14.696 psi, 1 atm = 101.325 kPa, and 1 atm = 1.01325 bar (inner cover page of text). Then the desired conversions become:

In atm:

$$P = (90 \text{ kgf/cm}^2) \left( \frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2} \right) = 87.1 \text{ atm}$$

In psi:

$$P = (90 \text{ kgf/cm}^2) \left( \frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2} \right) \left( \frac{14.696 \text{ psi}}{1 \text{ atm}} \right) = 1280 \text{ psi}$$

In kPa:

$$P = (90 \text{ kgf/cm}^2) \left( \frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2} \right) \left( \frac{101.325 \text{ kPa}}{1 \text{ atm}} \right) = 8826 \text{ kPa}$$

In bars:

$$P = (90 \text{ kgf/cm}^2) \left( \frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2} \right) \left( \frac{1.01325 \text{ bar}}{1 \text{ atm}} \right) = 88.3 \text{ bar}$$

Discussion

Note that the units atm, kgf/cm$^2$, and bar are almost identical to each other. The final results are given to three or four significant digits, but conversion ratios are typically precise to at least five significant digits.
Solution  A barometer is used to measure the altitude of a plane relative to the ground. The barometric readings at the ground and in the plane are given. The altitude of the plane is to be determined.

Assumptions  The variation of air density with altitude is negligible.

Properties  The densities of air and mercury are given to be $\rho_{\text{air}} = 1.20 \text{ kg/m}^3$ and $\rho_{\text{mercury}} = 13,600 \text{ kg/m}^3$.

Analysis  Atmospheric pressures at the location of the plane and the ground level are

$$
P_{\text{plane}} = (\rho g h)_{\text{plane}} = (13,600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.690 \text{ m}) = 92.06 \text{ kPa}
$$

$$
P_{\text{ground}} = (\rho g h)_{\text{ground}} = (13,600 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.753 \text{ m}) = 100.46 \text{ kPa}
$$

Taking an air column between the airplane and the ground and writing a force balance per unit base area, we obtain

$$
W_{\text{air}} \cdot A = P_{\text{ground}} - P_{\text{plane}}
$$

$$
(\rho g h)_{\text{air}} = P_{\text{ground}} - P_{\text{plane}}
$$

$$
(1.20 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(h)\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) = (100.46 - 92.06) \text{ kPa}
$$

It yields $h = 714 \text{ m}$, which is also the altitude of the airplane.

Discussion  Obviously, a mercury barometer is not practical on an airplane – an electronic barometer is used instead.

Solution  A 12-m high cylindrical container is filled with equal volumes of water and oil. The pressure difference between the top and the bottom of the container is to be determined.

Properties  The density of water is given to be $\rho = 1000 \text{ kg/m}^3$. The specific gravity of oil is given to be 0.85.

Analysis  The density of the oil is obtained by multiplying its specific gravity by the density of water,

$$
\rho_{\text{oil}} = \text{SG} \times \rho_{\text{water}} = (0.85)(1000 \text{ kg/m}^3) = 850 \text{ kg/m}^3
$$

The pressure difference between the top and the bottom of the cylinder is the sum of the pressure differences across the two fluids,

$$
\Delta P_{\text{total}} = \Delta P_{\text{oil}} + \Delta P_{\text{water}} = (\rho g h)_{\text{oil}} + (\rho g h)_{\text{water}}
$$

$$
= \left[850 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(6 \text{ m}) + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(6 \text{ m})\right] \left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right)
$$

$$
= 109 \text{ kPa}
$$

Discussion  The pressure at the interface must be the same in the oil and the water. Therefore, we can use the rules for hydrostatics across the two fluids, since they are at rest and there are no appreciable surface tension effects.
Chapter 3 Pressure and Fluid Statics

3-131
Solution The pressure of a gas contained in a vertical piston-cylinder device is measured to be 500 kPa. The mass of the piston is to be determined.

Assumptions There is no friction between the piston and the cylinder.

Analysis Drawing the free body diagram of the piston and balancing the vertical forces yield

\[ W = PA - P_{atm}A \]
\[ mg = (P - P_{atm})A \]
\[ m \left(9.81 \text{ m/s}^2\right) = (500 - 100 \text{ kPa}) \left(30 \times 10^{-4} \text{ m}^2\right) \left(\frac{1000 \text{ kg/m} \cdot \text{s}^2}{1 \text{ kPa}}\right) \]

Solution of the above equation yields \( m = 122 \text{ kg} \).

Discussion The gas cannot distinguish between pressure due to the piston weight and atmospheric pressure – both “feel” like a higher pressure acting on the top of the gas in the cylinder.

3-132
Solution The gage pressure in a pressure cooker is maintained constant at 100 kPa by a petcock. The mass of the petcock is to be determined.

Assumptions There is no blockage of the pressure release valve.

Analysis Atmospheric pressure is acting on all surfaces of the petcock, which balances itself out. Therefore, it can be disregarded in calculations if we use the gage pressure as the cooker pressure. A force balance on the petcock (\( \Sigma F_y = 0 \)) yields

\[ W = P_{gage}A \]
\[ m = \frac{P_{gage}A}{g} = \frac{(100 \text{ kPa})(4 \times 10^{-6} \text{ m}^2)}{9.81 \text{ m/s}^2} \left(\frac{1000 \text{ kg/m} \cdot \text{s}^2}{1 \text{ kPa}}\right) \]
\[ = 0.0408 \text{ kg} = 40.8 \text{ g} \]

Discussion The higher pressure causes water in the cooker to boil at a higher temperature.

3-133
Solution A glass tube open to the atmosphere is attached to a water pipe, and the pressure at the bottom of the tube is measured. It is to be determined how high the water will rise in the tube.

Properties The density of water is given to be \( \rho = 1000 \text{ kg/m}^3 \).

Analysis The pressure at the bottom of the tube can be expressed as

\[ P = P_{atm} + \left(\rho g h\right)_{\text{tube}} \]

Solving for \( h \),

\[ h = \frac{P - P_{atm}}{\rho g} \]
\[ = \frac{(115 - 98) \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left(\frac{1 \text{ kg} \cdot \text{m}^2 \cdot \text{s}^2}{1 \text{ N}}\right) \left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right) \]
\[ = 1.73 \text{ m} \]

Discussion Even though the water is flowing, the water in the tube itself is at rest. If the pressure at the tube bottom had been given in terms of gage pressure, we would not have had to take into account the atmospheric pressure term.
Solution  The average atmospheric pressure is given as \( P_{\text{atm}} = 101.325 (1 - 0.02256 z)^{5.256} \) where \( z \) is the altitude in km. The atmospheric pressures at various locations are to be determined.

Analysis  Atmospheric pressure at various locations is obtained by substituting the altitude \( z \) values in km into the relation \( P_{\text{atm}} = 101.325 (1 - 0.02256 z)^{5.256} \). The results are tabulated below.

- Atlanta: \( z = 0.306 \) km: \( P_{\text{atm}} = 101.325 (1 - 0.02256 \times 0.306)^{5.256} = 97.7 \) kPa
- Denver: \( z = 1.610 \) km: \( P_{\text{atm}} = 101.325 (1 - 0.02256 \times 1.610)^{5.256} = 83.4 \) kPa
- M. City: \( z = 2.309 \) km: \( P_{\text{atm}} = 101.325 (1 - 0.02256 \times 2.309)^{5.256} = 76.5 \) kPa
- Mt. Ev.: \( z = 8.848 \) km: \( P_{\text{atm}} = 101.325 (1 - 0.02256 \times 8.848)^{5.256} = 31.4 \) kPa

Discussion  It may be surprising, but the atmospheric pressure on Mt. Everest is less than 1/3 that at sea level!

Solution  The air pressure in a duct is measured by an inclined manometer. For a given vertical level difference, the gage pressure in the duct and the length of the differential fluid column are to be determined.

Assumptions  The manometer fluid is an incompressible substance.

Properties  The density of the liquid is given to be \( \rho = 0.81 \) kg/L = 810 kg/m³.

Analysis  The gage pressure in the duct is determined from

\[
P_{\text{gage}} = P_{\text{abs}} - P_{\text{atm}} = \rho gh
= (810 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.08 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ Pa}}{1 \text{ N/m}^2} \right)
= 636 \text{ Pa}
\]

The length of the differential fluid column is

\[
L = h / \sin \theta = (8 \text{ cm}) / \sin 35^\circ = 13.9 \text{ cm}
\]

Discussion  Note that the length of the differential fluid column is extended considerably by inclining the manometer arm for better readability (and therefore higher precision).
Solution

Equal volumes of water and oil are poured into a U-tube from different arms, and the oil side is pressurized until the contact surface of the two fluids moves to the bottom and the liquid levels in both arms become the same. The excess pressure applied on the oil side is to be determined.

Assumptions

1. Both water and oil are incompressible substances.
2. Oil does not mix with water.
3. The cross-sectional area of the U-tube is constant.

Properties

The density of oil is given to be $\rho_{oil} = 49.3 \text{ lbm/ft}^3$. We take the density of water to be $\rho_w = 62.4 \text{ lbm/ft}^3$.

Analysis

Noting that the pressure of both the water and the oil is the same at the contact surface, the pressure at this surface can be expressed as

$$P_{contact} = P_{\text{blow}} + \rho_w g h_a = P_{\text{atm}} + \rho_w g h_w$$

Noting that $h_a = h_w$ and rearranging,

$$P_{\text{gage, blow}} = P_{\text{blow}} - P_{\text{atm}} = (\rho_w - \rho_{oil}) g h
\begin{align*}
&= (62.4 \cdot 49.3 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(40/12 \text{ ft})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right)\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right) \\
&= 0.303 \text{ psi}
\end{align*}$$

Discussion

When the person stops blowing, the oil rises and some water flows into the right arm. It can be shown that when the curvature effects of the tube are disregarded, the differential height of water is 23.7 in to balance 30-in of oil.
Solution  A cylindrical container equipped with a manometer is inverted and pressed into water. The differential height of the manometer and the force needed to hold the container in place are to be determined.

Assumptions  1 Atmospheric pressure acts on all surfaces, and thus it can be ignored in calculations for convenience. 2 The variation of air pressure inside cylinder is negligible.

Properties  We take the density of water to be 1000 kg/m$^3$. The density of the manometer fluid is

\[
\rho_{\text{mano}} = \text{SG} \times \rho_w = 2.1 \left(1000 \text{ kg/m}^3\right) = 2100 \text{ kg/m}^3
\]

Analysis  The pressures at point $A$ and $B$ must be the same since they are on the same horizontal line in the same fluid. Then the gage pressure in the cylinder becomes

\[
P_{\text{air, gage}} = \rho_w g h_w = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.20 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 1962 \text{ N/m}^2 = 1962 \text{ Pa}
\]

The manometer also indicates the gage pressure in the cylinder. Therefore,

\[
P_{\text{air, gage}} = (\rho g h)_{\text{mano}} \rightarrow h = \frac{P_{\text{air, gage}}}{\rho_{\text{mano}} g} = \frac{1962 \text{ N/m}^2}{2100 \text{ kg/m}^3(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kN/m}^2}{1 \text{ kN/m}^2} \right) = 0.0950 \text{ m} = 9.50 \text{ cm}
\]

A force balance on the cylinder in the vertical direction yields

\[
F + W = P_{\text{air, gage}} A_c
\]

Solving for $F$ and substituting,

\[
F = P_{\text{air, gage}} \frac{\pi D^2}{4} - W = (1962 \text{ N/m}^2) \frac{\pi (0.25 \text{ m})^2}{4} - 65 \text{ N} = 31.3 \text{ N}
\]

Discussion  We could also solve this problem by considering the atmospheric pressure, but we would obtain the same result since atmospheric pressure would cancel out.
Solution

A gasoline line is connected to a pressure gage through a double-U manometer. For a given reading of the pressure gage, the gage pressure of the gasoline line is to be determined.

Assumptions

1. All the liquids are incompressible.
2. The effect of air column on pressure is negligible.

Properties

The specific gravities of oil, mercury, and gasoline are given to be 0.79, 13.6, and 0.70, respectively. We take the density of water to be \( \rho_w = 1000 \text{ kg/m}^3 \).

Analysis

Starting with the pressure indicated by the pressure gage and moving along the tube by adding (as we go down) or subtracting (as we go up) the \( \rho g h \) terms until we reach the gasoline pipe, and setting the result equal to \( P_{\text{gasoline}} \) gives

\[
P_{\text{gage}} - \rho_w gh_w + \rho_{\text{oil}} gh_{\text{oil}} - \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{gasoline}} gh_{\text{gasoline}} = P_{\text{gasoline}}
\]

Rearranging,

\[
P_{\text{gasoline}} = P_{\text{gage}} - \rho_w g (h_w - S_{\text{oil}} h_{\text{oil}} + S_{\text{Hg}} h_{\text{Hg}} + S_{\text{gasoline}} h_{\text{gasoline}})
\]

Substituting,

\[
P_{\text{gasoline}} = 370 \text{ kPa} - \left( \frac{1000 \text{ kg/m}^3}{1000 \text{ kg/m}^2/s^2} \right) \left( \frac{1 \text{ kPa}}{1 \text{ kN/m}^2} \right) \left( 0.45 \text{ m} - 0.79(0.5 \text{ m}) + 13.6(0.1 \text{ m}) + 0.70(0.22 \text{ m}) \right)
\]

\[
= 354.6 \text{ kPa} \approx 355 \text{ kPa}
\]

Therefore, the pressure in the gasoline pipe is 15.4 kPa lower than the pressure reading of the pressure gage.

Discussion

Note that sometimes the use of specific gravity offers great convenience in the solution of problems that involve several fluids.
A gasoline line is connected to a pressure gage through a double-U manometer. For a given reading of the pressure gage, the gage pressure of the gasoline line is to be determined.

Assumptions
1. All the liquids are incompressible.
2. The effect of air column on pressure is negligible.

Properties
The specific gravities of oil, mercury, and gasoline are given to be 0.79, 13.6, and 0.70, respectively. We take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$.

Analysis
Starting with the pressure indicated by the pressure gage and moving along the tube by adding (as we go down) or subtracting (as we go up) the $\rho gh$ terms until we reach the gasoline pipe, and setting the result equal to $P_{\text{gasoline}}$ gives

$$P_{\text{gage}} - \rho_w g h_w + \rho_{\text{alcohol}} g h_{\text{alcohol}} - \rho_{\text{Hg}} g h_{\text{Hg}} - \rho_{\text{gasoline}} g h_{\text{gasoline}} = P_{\text{gasoline}}$$

Rearranging,

$$P_{\text{gasoline}} = P_{\text{gage}} - \rho_w g (h_w - SG_{\text{alcohol}} h_{\text{alcohol}} + SG_{\text{Hg}} h_{\text{Hg}} + SG_{\text{gasoline}} h_{\text{gasoline}})$$

Substituting,

$$P_{\text{gasoline}} = 280 \text{ kPa} - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(0.45 \text{ m}) - 0.79(0.5 \text{ m}) + 13.6(0.1 \text{ m}) + 0.70(0.22 \text{ m})]$$

$$= 264.6 \text{ kPa} \equiv 265 \text{ kPa}$$

Therefore, the pressure in the gasoline pipe is 15.4 kPa lower than the pressure reading of the pressure gage.

Discussion
Note that sometimes the use of specific gravity offers great convenience in the solution of problems that involve several fluids.
3-140E

Solution  A water pipe is connected to a double-U manometer whose free arm is open to the atmosphere. The absolute pressure at the center of the pipe is to be determined.

Assumptions  1 All the liquids are incompressible. 2 The solubility of the liquids in each other is negligible.

Properties  The specific gravities of mercury and oil are given to be 13.6 and 0.80, respectively. We take the density of water to be $\rho_w = 62.4 \text{ lbm/ft}^3$.

Analysis  Starting with the pressure at the center of the water pipe, and moving along the tube by adding (as we go down) or subtracting (as we go up) the $\rho g h$ terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to $P_{atm}$ gives

$$P_{water \ pipe} - \rho_{water} g h_{water} + \rho_{alcohol} g h_{alcohol} - \rho_{Hg} g h_{Hg} - \rho_{oil} g h_{oil} = P_{atm}$$

Solving for $P_{water \ pipe}$,

$$P_{water \ pipe} = P_{atm} + \rho_{water} g (h_{water} - S G_{oil} h_{alcohol} + S G_{Hg} h_{Hg} + S G_{oil} h_{oil})$$

Substituting,

$$P_{water \ pipe} = 14.2 \text{ psia} + (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/lb}^2)((35/12 \text{ ft}) - 0.80(60/12 \text{ ft}) + 13.6(15/12 \text{ ft}))$$

$$+ 0.8(40/12 \text{ ft}) \times \left( \frac{1 \text{ lb}^2}{32.2 \text{ lbm} \cdot \text{ ft}^2} \right) \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right)$$

$$= 22.3 \text{ psia}$$

Therefore, the absolute pressure in the water pipe is 22.3 psia.

Discussion  Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly.
3-141

Solution  The pressure of water flowing through a pipe is measured by an arrangement that involves both a pressure gage and a manometer. For the values given, the pressure in the pipe is to be determined.

Assumptions  1 All the liquids are incompressible.  2 The effect of air column on pressure is negligible.

Properties  The specific gravity of gage fluid is given to be 2.4. We take the standard density of water to be $\rho_w = 1000$ kg/m$^3$.

Analysis  Starting with the pressure indicated by the pressure gage and moving along the tube by adding (as we go down) or subtracting (as we go up) the $\rho g h$ terms until we reach the water pipe, and setting the result equal to $P_{\text{water}}$ give

$$P_{\text{gage}} + \rho_w g h_{w1} - \rho_{\text{gage}} g h_{\text{gage}} - \rho_w g h_{w2} = P_{\text{water}}$$

Rearranging,

$$P_{\text{water}} = P_{\text{gage}} + \rho_w g (h_{w1} - S G_{\text{gage}} h_{\text{gage}} - h_{w2}) = P_{\text{gage}} + \rho_w g (h_2 - S G_{\text{gage}} L_2 \sin \theta - L_2 \sin \theta)$$

Noting that $\sin \theta = \frac{8}{12} = 0.6667$ and substituting,

$$P_{\text{water}} = 30 \text{ kPa} + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.50 \text{ m}) - 2.4(0.06 \text{ m})0.6667 - (0.06 \text{ m})0.6667$$

$$= 33.6 \text{ kPa}$$

Therefore, the pressure in the gasoline pipe is 3.6 kPa over the reading of the pressure gage.

Discussion  Note that even without a manometer, the reading of a pressure gage can be in error if it is not placed at the same level as the pipe when the fluid is a liquid.
**Chapter 3 Pressure and Fluid Statics**

**Solution** A U-tube filled with mercury except the 12-cm high portion at the top. Oil is poured into the left arm, forcing some mercury from the left arm into the right one. The maximum amount of oil that can be added into the left arm is to be determined.

**Assumptions** 1 Both liquids are incompressible. 2 The U-tube is perfectly vertical.

**Properties** The specific gravities are given to be 2.72 for oil and 13.6 for mercury.

**Analysis** Initially, the mercury levels in both tubes are the same. When oil is poured into the left arm, it will push the mercury in the left down, which will cause the mercury level in the right arm to rise. Noting that the volume of mercury is constant, the decrease in the mercury volume in left column must be equal to the increase in the mercury volume in the right arm. Therefore, if the drop in mercury level in the left arm is \( x \), the rise in the mercury level in the right arm \( h \) corresponding to a drop of \( x \) in the left arm is

\[
V_{\text{left}} = V_{\text{right}} \quad \rightarrow \quad \pi(2d)^2 x = \pi d^2 h \quad \rightarrow \quad h = 4x
\]

The pressures at points \( A \) and \( B \) are equal \( P_A = P_B \) and thus

\[
P_{\text{air}} + \rho_{\text{oil}} g (h_{\text{oil}} + x) = P_{\text{atm}} + \rho_{\text{Hg}} g h_{\text{Hg}} \quad \rightarrow \quad \rho_{\text{oil}} g (h_{\text{oil}} + x) = \rho_{\text{Hg}} g h_{\text{Hg}}
\]

Solving for \( x \) and substituting,

\[
x = \frac{\frac{\rho_{\text{Hg}} g h_{\text{Hg}}}{\rho_{\text{oil}} g (h_{\text{oil}} + x)}}{5 \times 13.6 - 2.72} = 0.5 \text{ cm}
\]

Therefore, the maximum amount of oil that can be added into the left arm is

\[
V_{\text{oil, max}} = \pi(2d/2)^2 (h_{\text{oil}} + x) = \pi (2 \text{ cm})^2 (12 \text{ cm} + 0.5 \text{ cm}) = 157.08 \text{ cm}^3 \approx 157 \text{ cm}^3 = 0.157 \text{ L}
\]

**Discussion** Note that the fluid levels in the two arms of a U-tube can be different when two different fluids are involved.
Solution  The pressure buildup in a teapot may cause the water to overflow through the service tube. The maximum cold-water height to avoid overflow under a specified gage pressure is to be determined.

Assumptions  1 Water is incompressible.  2 Thermal expansion and the amount of water in the service tube are negligible.  3 The cold water temperature is 20°C.

Properties  The density of water at 20°C is $\rho_w = 998.0 \text{ kg/m}^3$.

Analysis  From geometric considerations, the vertical distance between the bottom of the teapot and the tip of the service tube is

$$h_{\text{tip}} = 3 + 12 \cos 35° = 12.8 \text{ cm}$$

This would be the maximum water height if there were no pressure build-up inside by the steam. The steam pressure inside the teapot above the atmospheric pressure must be balanced by the water column inside the service tube,

$$P_{v, \text{gage}} = \rho_w g \Delta h_w$$

or,

$$\Delta h_w = \frac{P_{v, \text{gage}}}{\rho_w g} = \frac{0.32 \text{ kPa}}{(998.0 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kN/m}^2} \right) \left( \frac{1 \text{ kN}}{1 \text{ kPa}} \right) = 0.033 \text{ m} = 3.3 \text{ cm}$$

Therefore, the water level inside the teapot must be 3.3 cm below the tip of the service tube. Then the maximum initial water height inside the teapot to avoid overflow becomes

$$h_{w, \text{max}} = h_{\text{tip}} - \Delta h_w = 12.8 - 3.3 = 9.5 \text{ cm}$$

Discussion  We can obtain the same result formally by starting with the vapor pressure in the teapot and moving along the service tube by adding (as we go down) or subtracting (as we go up) the $\rho gh$ terms until we reach the atmosphere, and setting the result equal to $P_{\text{atm}}$:

$$P_{\text{atm}} + P_{v, \text{gage}} - \rho_w gh_w = P_{\text{atm}} \rightarrow P_{v, \text{gage}} = \rho_w gh_w$$
Solution The pressure buildup in a teapot may cause the water to overflow through the service tube. The maximum cold-water height to avoid overflow under a specified gage pressure is to be determined by considering the effect of thermal expansion.

Assumptions 1 The amount of water in the service tube is negligible. 3 The cold water temperature is 20°C.

Properties The density of water is \( \rho_w = 998.0 \text{ kg/m}^3 \) at 20°C, and \( \rho_w = 957.9 \text{ kg/m}^3 \) at 100°C

Analysis From geometric considerations, the vertical distance between the bottom of the teapot and the tip of the service tube is

\[
h_{\text{tip}} = 4 + 12 \cos 40^\circ = 13.2 \text{ cm}
\]

This would be the maximum water height if there were no pressure build-up inside by the steam. The steam pressure inside the teapot above the atmospheric pressure must be balanced by the water column inside the service tube,

\[
P_{v,\text{gage}} = \rho_w g \Delta h_w
\]

or,

\[
\Delta h_w = \frac{P_{v,\text{gage}}}{\rho_w g} = \frac{0.32 \text{ kPa}}{998.0 \text{ kg/m}^3 \times (9.81 \text{ m/s}^2)} \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kN}} \right) \left( \frac{1 \text{ kN/m}^2}{1 \text{ kPa}} \right) = 0.033 \text{ m} = 3.3 \text{ cm}
\]

Therefore, the water level inside the teapot must be 3.4 cm below the tip of the service tube. Then the height of hot water inside the teapot to avoid overflow becomes

\[
h_w = h_{\text{tip}} - \Delta h_w = 13.2 - 3.4 = 9.8 \text{ cm}
\]

The specific volume of water is \( 1/998 \text{ m}^3/\text{kg} \) at 20°C and \( 1/957.9 \text{ m}^3/\text{kg} \) at 100°C. Then the percent drop in the volume of water as it cools from 100°C to 20°C is

\[
\text{Volume reduction} = \frac{\nu_{100^\circ C} - \nu_{20^\circ C}}{\nu_{100^\circ C}} = \frac{1/957.9 - 1/998.0}{1/957.9} = 0.040 \text{ or 4.0%}
\]

Volume is proportional to water height, and to allow for thermal expansion, the volume of cold water should be 4% less. Therefore, the maximum initial water height to avoid overflow should be

\[
h_{w,\text{max}} = (1 - 0.040)h_w = 0.96 \times 9.8 \text{ cm} = 9.4 \text{ cm}
\]

Discussion Note that the effect of thermal expansion can be quite significant.
Solution  The temperature of the atmosphere varies with altitude \( z \) as \( T = T_0 - \beta z \), while the gravitational acceleration varies by \( g(z) = g_0 / (1 + z / 6,370,320)^2 \). Relations for the variation of pressure in atmosphere are to be obtained (a) by ignoring and (b) by considering the variation of \( g \) with altitude.

Assumptions  The air in the troposphere behaves as an ideal gas.

Analysis  
(a) Pressure change across a differential fluid layer of thickness \( dz \) in the vertical \( z \) direction is 
\[
dP = -\rho g dz
\]
From the ideal gas relation, the air density can be expressed as \( \rho = \frac{P}{RT} = \frac{P}{R(T_0 - \beta z)} \). Then,
\[
dP = -\frac{P}{R(T_0 - \beta z)} \rho g dz
\]
Separating variables and integrating from \( z = 0 \) where \( P = P_0 \) to \( z = z \) where \( P = P \),
\[
\int_{P_0}^{P} \frac{dP}{P} = -\frac{g}{RT} \int_0^z \frac{dz}{R(T_0 - \beta z)}
\]
Performing the integrations,
\[
\ln \frac{P}{P_0} = \frac{g}{R \beta} \ln \frac{T_0 - \beta z}{T_0}
\]
Rearranging, the desired relation for atmospheric pressure for the case of constant \( g \) becomes
\[
P = P_0 \left(1 - \frac{\beta z}{T_0} \right)^{\frac{g}{R \beta}}
\]
(b) When the variation of \( g \) with altitude is considered, the procedure remains the same but the expressions become more complicated,
\[
dP = -\frac{P}{R(T_0 - \beta z)} \left(1 + \frac{z}{6,370,320}\right)^{-2} dz
\]
Separating variables and integrating from \( z = 0 \) where \( P = P_0 \) to \( z = z \) where \( P = P \),
\[
\int_{P_0}^{P} \frac{dP}{P} = -\frac{g_0 dz}{R(T_0 - \beta z)(1 + z / 6,370,320)^2}
\]
Performing the integrations,
\[
\ln P \bigg|_{P_0}^{P} = \frac{g_0}{R \beta} \left[ \frac{1}{1 + kT_0 / \beta} \ln \frac{1 + kz}{1 + kT_0 / \beta} - \frac{1}{1 + kT_0 / \beta} \ln \frac{1 + kT_0}{1 + kT_0 / \beta} \right]_{0}^{z}
\]
where \( R = 287 \text{ J/kg·K} = 287 \text{ m}^2/\text{s}^2·\text{K} \) is the gas constant of air. After some manipulations, we obtain
\[
P = P_0 \exp \left[ -\frac{g_0}{R(\beta + kT_0)} \left( \frac{1}{1 + 1/kz} + \frac{1}{1 + kT_0 / \beta} \ln \frac{1 + kT_0}{1 - \beta z / T_0} \right) \right]
\]
where \( T_0 = 288.15 \text{ K} \), \( \beta = 0.0065 \text{ K/m} \), \( g_0 = 9.807 \text{ m/s}^2 \), \( k = 1/6,370,320 \text{ m}^{-1} \), and \( z \) is the elevation in m.

Discussion  When performing the integration in part (b), the following expression from integral tables is used, together with a transformation of variable \( x = T_0 - \beta z \),
\[
\int_{x=a}^{x} \frac{dx}{x(a+bx)^2} = \frac{1}{a(bx)} - \frac{1}{a^2} \ln \frac{a+bx}{x}
\]
Also, for \( z = 11,000 \text{ m} \), for example, the relations in (a) and (b) give 22.62 and 22.69 kPa, respectively.
Solution  The variation of pressure with density in a thick gas layer is given. A relation is to be obtained for pressure as a function of elevation $z$.

Assumptions  The property relation $P = C \rho^n$ is valid over the entire region considered.

Analysis  The pressure change across a differential fluid layer of thickness $dz$ in the vertical $z$ direction is given as,

$$dP = -\rho g dz$$

Also, the relation $P = C \rho^n$ can be expressed as $C = P / \rho^n = P_0 / \rho_0^n$, and thus

$$\rho = \rho_0 (P / P_0)^{1/n}$$

Substituting,

$$dP = -g \rho_0 (P / P_0)^{1/n} dz$$

Separating variables and integrating from $z = 0$ where $P = P_0 = C \rho_0^n$ to $z = z$ where $P = P$,

$$\int_{P_0}^{P} (P / P_0)^{-1/n} dP = -\rho_0 g \int_{0}^{z} dz$$

Performing the integrations.

$$P_0 \left( \frac{P_0}{P} \right)^{-1/n + 1} \rho_0 = -\rho_0 g z \quad \rightarrow \quad \left( \frac{P}{P_0} \right)^{(n-1)/n} - 1 = -\frac{n-1}{n} \frac{\rho_0 g z}{P_0}$$

Solving for $P$,

$$P = P_0 \left( 1 - \frac{n-1}{n} \frac{\rho_0 g z}{P_0} \right)^{n/(n-1)}$$

which is the desired relation.

Discussion  The final result could be expressed in various forms. The form given is very convenient for calculations as it facilitates unit cancellations and reduces the chance of error.
Solution  A pressure transducer is used to measure pressure by generating analogue signals, and it is to be calibrated by measuring both the pressure and the electric current simultaneously for various settings, and the results are to be tabulated. A calibration curve in the form of \( P = al + b \) is to be obtained, and the pressure corresponding to a signal of 10 mA is to be calculated.

Assumptions  Mercury is an incompressible liquid.

Properties  The specific gravity of mercury is given to be 13.56, and thus its density is 13,560 kg/m³.

Analysis  For a given differential height, the pressure can be calculated from

\[
P = \rho g \Delta h
\]

For \( \Delta h = 28.0 \text{ mm} = 0.0280 \text{ m} \), for example,

\[
P = 13.56(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.0280 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1 \text{ kN/m}^2}\right) = 3.72 \text{ kPa}
\]

Repeating the calculations and tabulating, we have

<table>
<thead>
<tr>
<th>( \Delta h ) (mm)</th>
<th>28.0</th>
<th>181.5</th>
<th>297.8</th>
<th>413.1</th>
<th>765.9</th>
<th>1027</th>
<th>1149</th>
<th>1362</th>
<th>1458</th>
<th>1536</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P ) (kPa)</td>
<td>3.72</td>
<td>24.14</td>
<td>39.61</td>
<td>54.95</td>
<td>101.9</td>
<td>136.6</td>
<td>152.8</td>
<td>181.2</td>
<td>193.9</td>
<td>204.3</td>
</tr>
<tr>
<td>( I ) (mA)</td>
<td>4.21</td>
<td>5.78</td>
<td>6.97</td>
<td>8.15</td>
<td>11.76</td>
<td>14.43</td>
<td>15.68</td>
<td>17.86</td>
<td>18.84</td>
<td>19.64</td>
</tr>
</tbody>
</table>

A plot of \( P \) versus \( I \) is given below. It is clear that the pressure varies linearly with the current, and using EES, the best curve fit is obtained to be

\[
P = 13.00I - 51.00 \quad (\text{kPa}) \quad \text{for} \quad 4.21 \leq I \leq 19.64.
\]

For \( I = 10 \text{ mA} \), for example, we would get \( P = 79.0 \text{ kPa} \).

Discussion  Note that the calibration relation is valid in the specified range of currents or pressures.
Solution  A system is equipped with two pressure gages and a manometer. For a given differential fluid height, the pressure difference \( \Delta P = P_2 - P_1 \) is to be determined.

Assumptions  1 All the liquids are incompressible.  2 The effect of air column on pressure is negligible.

Properties  The specific gravities are given tone 2.15 for the gage fluid and 0.87 for oil. We take the standard density of water to be \( \rho_w = 1000 \text{ kg/m}^3 \).

Analysis  Starting with the pressure indicated by the pressure gage 2 and moving along the tube by adding (as we go down) or subtracting (as we go up) the \( \rho gh \) terms and ignoring the air spaces until we reach the pressure gage 1, and setting the result equal to \( P_1 \) give

\[
P_2 - \rho_{\text{gage}} g \Delta h + \rho_{\text{oil}} g h_{\text{oil}} = P_1
\]

Rearranging,

\[
P_2 - P_1 = \rho_w g \left( \text{SG}_{\text{gage}} \Delta h - \text{SG}_{\text{oil}} h_{\text{oil}} \right)
\]

Substituting,

\[
P_2 - P_1 = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.15(0.12 \text{ m}) - 0.87(0.65 \text{ m}))
\]

\[
= -3.02 \text{ kPa}
\]

Therefore, the pressure reading of the left gage is 3.02 kPa lower than that of the right gage.

Discussion  The negative pressure difference indicates that the pressure differential across the oil level is greater than the pressure differential corresponding to the differential height of the manometer fluid.
Solution  An oil pipeline and a rigid air tank are connected to each other by a manometer. The pressure in the pipeline and the change in the level of manometer fluid due to a air temperature drop are to be determined.

Assumptions  1 All the liquids are incompressible. 2 The effect of air column on pressure is negligible. 3 The air volume in the manometer is negligible compared with the volume of the tank.

Properties  The specific gravities are given to be 2.68 for oil and 13.6 for mercury. We take the standard density of water to be $\rho_w = 1000 \text{ kg/m}^3$. The gas constant of air is $0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$.

Analysis  (a) Starting with the oil pipe and moving along the tube by adding (as we go down) or subtracting (as we go up) the $\rho g h$ terms until we reach the air tank, and setting the result equal to $P_{air}$ give

$$P_{oil} + \rho_{oil} g h_{oil} + \rho_{Hg} g h_{Hg} = P_{air}$$

The absolute pressure in the air tank is determined from the ideal-gas relation $PV = mRT$ to be

$$P_{air} = \frac{mRT}{V} = \frac{(15 \text{ kg})(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(80 + 273)\text{K}}{1.3 \text{ m}^3} = 1169 \text{ kPa}$$

Then the absolute pressure in the oil pipe becomes

$$P_{oil} = P_{air} - \rho_{air} g h_{air} - \rho_{Hg} g h_{Hg}$$

$$= 1169 \text{ kPa} - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.68(0.75 \text{ m}) + 13.6(0.20 \text{ m}))\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1 \text{ kN/m}^2}\right)$$

$$= 1123 \text{ kPa} \approx 1120 \text{ kPa}$$

(b) The pressure in the air tank when the temperature drops to 20°C becomes

$$P_{air} = \frac{mRT}{V} = \frac{(15 \text{ kg})(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(20 + 273)\text{K}}{1.3 \text{ m}^3} = 970 \text{ kPa}$$

When the mercury level in the left arm drops a distance $x$, the rise in the mercury level in the right arm $y$ becomes

$$\nu_{left} = \nu_{right} \quad \Rightarrow \quad \pi(3d)^2 x = \pi d^2 y \quad \Rightarrow \quad y = 9x \quad \text{and} \quad y_{vert} = 9x \sin 50^\circ$$

and the mercury fluid height will change by $x + 9x \sin 50^\circ$ or 7.894x. Then,

$$P_{oil} + \rho_{oil} g(h_{oil} + x) + \rho_{Hg} g(h_{Hg} - 7.894x) = P_{air} \quad \Rightarrow \quad \text{SG}_{oil}(h_{oil} + x) + \text{SG}_{Hg}(h_{Hg} - 7.894x) = \frac{P_{air} - P_{oil}}{\rho_{air} g}$$

Substituting,

$$2.68(0.75 + x) + 13.6(0.20 - 7.894x) = \frac{(970 - 1123) \text{ kPa}}{1000 \text{ kg/m}^3}(9.81 \text{ m/s}^2)\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1 \text{ kN/m}^2}\right)$$

which yields $x = 0.194 \text{ m} = 19.4 \text{ cm}$. Therefore, the oil-mercury interface will drop 19.4 cm as a result of the temperature drop of air.

Discussion  Note that the pressure in constant-volume gas chambers is very sensitive to temperature changes.
Solution  The density of a wood log is to be measured by tying lead weights to it until both the log and the weights are completely submerged, and then weighing them separately in air. The average density of a given log is to be determined by this approach.

Properties  The density of lead weights is given to be 11,300 kg/m³. We take the density of water to be 1000 kg/m³.

Analysis  The weight of a body is equal to the buoyant force when the body is floating in a fluid while being completely submerged in it (a consequence of vertical force balance from static equilibrium). In this case the average density of the body must be equal to the density of the fluid since

\[ W = F_B \rightarrow \rho_{\text{body}} g V = \rho_{\text{fluid}} g V \rightarrow \rho_{\text{body}} = \rho_{\text{fluid}} \]

Therefore,

\[ \rho_{\text{ave}} = \frac{m_{\text{total}}}{V_{\text{total}}} = \frac{m_{\text{lead}} + m_{\log}}{V_{\text{lead}} + V_{\log}} = \rho_{\text{water}} \rightarrow V_{\log} = \frac{m_{\text{lead}} + m_{\log}}{\rho_{\text{water}}} - V_{\text{lead}} \]

where

\[ V_{\text{lead}} = \frac{m_{\text{lead}}}{\rho_{\text{lead}}} = \frac{34 \text{ kg}}{11,300 \text{ kg/m}^3} = 3.0089 \times 10^{-3} \text{ m}^3 \]

\[ m_{\log} = \frac{W_{\log}}{g} = \frac{1540 \text{ N}}{9.807 \text{ m/s}^2} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) = 157.031 \text{ kg} \]

Substituting, the volume and density of the log are determined to be

\[ V_{\log} = \frac{m_{\text{lead}} + m_{\log}}{\rho_{\text{water}}} - V_{\text{lead}} = \frac{(34 + 157.031) \text{ kg}}{1000 \text{ kg/m}^3} - 3.0089 \times 10^{-3} \text{ m}^3 = 0.18802 \text{ m}^3 \]

\[ \rho_{\log} = \frac{m_{\log}}{V_{\log}} = \frac{157.031 \text{ kg}}{0.18802 \text{ m}^3} = 835.174 \text{ kg/m}^3 \approx 835 \text{ kg/m}^3 \]

Discussion  Note that the log must be completely submerged for this analysis to be valid. Ideally, the lead weights must also be completely submerged, but this is not very critical because of the small volume of the lead weights.
Solution

A rectangular gate that leans against the floor with an angle of 45° with the horizontal is to be opened from its lower edge by applying a normal force at its center. The minimum force \( F \) required to open the water gate is to be determined.

**Assumptions**
1. Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.
2. Friction at the hinge is negligible.

**Properties**
We take the density of water to be 1000 kg/m³ throughout.

**Analysis**

The length of the gate and the distance of the upper edge of the gate (point \( B \)) from the free surface in the plane of the gate are

\[
b = \frac{3 \text{ m}}{\sin 45°} = 4.243 \text{ m} \quad \text{and} \quad s = \frac{0.5 \text{ m}}{\sin 45°} = 0.7071 \text{ m}
\]

\[
h_c = \frac{h}{2} + 0.5 = \frac{3 \text{ m}}{2} + 0.5 \text{ m} = 2 \text{ m}
\]

The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic force on the surface,

\[
F_R = P_{ave} A = \rho g h_c A
\]

\[
= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2 \text{ m})[6 \times 4.243 \text{ m}^2] \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right)
\]

\[
= 499.5 \text{ kN}
\]

The distance of the pressure center from the free surface of water along the plane of the gate is

\[
y_p = s + \frac{b^2}{12(s+b/2)} = 0.7071 + \frac{4.243^2}{2} + \frac{4.243^2}{12(0.7071+4.243/2)} = 3.359 \text{ m}
\]

The distance of the pressure center from the hinge at point \( B \) is

\[
L_P = y_p - s = 3.359 - 0.7071 = 2.652 \text{ m}
\]

Taking the moment about point \( B \) and setting it equal to zero gives

\[
\sum M_B = 0 \quad \rightarrow \quad F_R L_P = Fb / 2
\]

Solving for \( F \) and substituting, the required force to overcome the pressure is

\[
F = \frac{2F_R L_P}{b} = \frac{2(499.5 \text{ kN})(2.652 \text{ m})}{4.243 \text{ m}} = 624.4 \text{ kN}
\]

In addition to this, there is the weight of the gate itself, which must be added. In the 45° direction,

\[
F_{\text{gate}} = W \cos(45°) = mg \cos(45°) = (280 \text{ kg})(9.81 \text{ m/s}^2) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \cos(45°) = 1.942 \text{ kN}
\]

Thus, the total force required in the 45° direction is the sum of these two values,

\[
F_{\text{total}} = 624.4 + 1.942 = 626.3 \text{ kN} \approx 626 \text{ kN} \text{ in the 45° direction}
\]

**Discussion**
The applied force is inversely proportional to the distance of the point of application from the hinge, and the required force can be reduced by applying the force at a lower point on the gate. The weight of the gate is nearly negligible compared to the pressure force in this example; in reality, a heavier gate would probably be required.
Solution  A rectangular gate that leans against the floor with an angle of 45° with the horizontal is to be opened from its lower edge by applying a normal force at its center. The minimum force $F$ required to open the water gate is to be determined.

Assumptions  1 Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.  2 Friction at the hinge is negligible.

Properties  We take the density of water to be 1000 kg/m$^3$ throughout.

Analysis  The length of the gate and the distance of the upper edge of the gate (point $B$) from the free surface in the plane of the gate are

$$ b = \frac{3 \text{ m}}{\sin 45°} = 4.243 \text{ m} \quad \text{and} \quad s = \frac{1.2 \text{ m}}{\sin 45°} = 1.697 \text{ m} $$

$$ h_C = \frac{h}{2} + 0.5 = \frac{3 \text{ m}}{2} + 1.2 \text{ m} = 2.7 \text{ m} $$

The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic on the surface,

$$ F_R = P_{ave}A = \rho gh_C A = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.7 \text{ m})[6 \times 4.243 \text{ m}^2 \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right)] $$

$$ = 674.3 \text{ kN} $$

The distance of the pressure center from the free surface of water along the plane of the gate is

$$ y_p = s + \frac{b}{2} = \frac{b^2}{2(2s+b/2)} = 1.697 + \frac{4.243}{2} = \frac{4.243^2}{2(1.697 + 4.243/2)} = 4.211 \text{ m} $$

The distance of the pressure center from the hinge at point $B$ is

$$ L_P = y_p - s = 4.211 - 1.697 = 2.514 \text{ m} $$

Taking the moment about point $B$ and setting it equal to zero gives

$$ \sum M_B = 0 \quad \rightarrow \quad F_R L_P = Fb / 2 $$

Solving for $F$ and substituting, the required force to overcome the pressure is

$$ F = \frac{2F_R L_P}{b} = \frac{2(674.3 \text{ kN})(2.514 \text{ m})}{4.243 \text{ m}} = 799.1 \text{ kN} $$

In addition to this, there is the weight of the gate itself, which must be added. In the 45° direction,

$$ F_{gate} = W \cos(45°) = mg \cos(45°) = (280 \text{ kg})(9.81 \text{ m/s}^2) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \cos(45°) = 1.942 \text{ kN} $$

Thus, the total force required in the 45° direction is the sum of these two values,

$$ F_{total} = 799.1 + 1.942 = 801.1 \text{ kN} \approx 801 \text{kN} \text{ in the 45° direction} $$

Discussion  The applied force is inversely proportional to the distance of the point of application from the hinge, and the required force can be reduced by applying the force at a lower point on the gate. The weight of the gate is nearly negligible compared to the pressure force in this example; in reality, a heavier gate would probably be required.
Solution  A rectangular gate hinged about a horizontal axis along its upper edge is restrained by a fixed ridge at point B. The force exerted to the plate by the ridge is to be determined.

Assumptions  Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.

Properties  We take the density of water to be 1000 kg/m\(^3\) throughout.

Analysis  The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic force on the gate,

\[
F_R = P_{\text{avg}}A = \rho g h_c A
\]

\[
= \left(1000 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)(3.5 \text{ m})\left(3 \times 6 \text{ m}^2\right)\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)
\]

\[
= 618 \text{ kN}
\]

The vertical distance of the pressure center from the free surface of water is

\[
y_p = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 2 + \frac{3}{2} + \frac{3^2}{12(2 + 3/2)} = 3.71 \text{ m}
\]

Discussion  You can calculate the force at point B required to hold back the gate by setting the net moment around hinge point A to zero.

---

Solution  A rectangular gate hinged about a horizontal axis along its upper edge is restrained by a fixed ridge at point B. The force exerted to the plate by the ridge is to be determined.

Assumptions  Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.

Properties  We take the density of water to be 1000 kg/m\(^3\) throughout.

Analysis  The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the wetted plate area gives the resultant hydrostatic force on the gate,

\[
F_R = P_{\text{avg}}A = \rho g h_c A
\]

\[
= \left(1000 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)(1 \text{ m})\left(2 \times 6 \text{ m}^2\right)\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)
\]

\[
= 118 \text{ kN}
\]

The vertical distance of the pressure center from the free surface of water is

\[
y_p = \frac{2h}{3} = \frac{2(2 \text{ m})}{3} = 1.33 \text{ m}
\]

Discussion  Compared to the previous problem (with higher water depth), the force is much smaller, as expected. Also, the center of pressure on the gate is much lower (closer to the ground) for the case with the lower water depth.
Solution  A semicircular tunnel is to be built under a lake. The total hydrostatic force acting on the roof of the tunnel is to be determined.

Assumptions  Atmospheric pressure acts on both sides of the tunnel, and thus it can be ignored in calculations for convenience.

Properties  We take the density of water to be 62.4 lbm/ft³ throughout.

Analysis  We consider the free body diagram of the liquid block enclosed by the circular surface of the tunnel and its vertical (on both sides) and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface (each side):
\[ F_H = F_x = P_{\text{ave}}A = \rho gh c A = \rho g(s + R/2)A \]
\[ = (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(130 + 20/2 \text{ ft})(20 \text{ ft} \times 800 \text{ ft}) \frac{1 \text{lbf}}{32.2 \text{ lbm} \cdot \text{ft}^2} \]
\[ = 1.398 \times 10^8 \text{ lbf} \text{ (on each side of the tunnel)} \]

Vertical force on horizontal surface (downward):
\[ F_y = P_{\text{ave}}A = \rho gh c A = \rho gh_{\text{top}}A \]
\[ = (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(130 \text{ ft})(40 \text{ ft} \times 800 \text{ ft}) \frac{1 \text{lbf}}{32.2 \text{ lbm} \cdot \text{ft}^2} \]
\[ = 2.596 \times 10^8 \text{ lbf} \]

Weight of fluid block on each side within the control volume (downward):
\[ W = mg = \rho g V = \rho g(R^2 - \pi R^2/4)(2000 \text{ ft}) \]
\[ = (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(20 \text{ ft}^2)(1 - \pi/4)(800 \text{ ft}) \frac{1 \text{lbf}}{32.2 \text{ lbm} \cdot \text{ft}^2} \]
\[ = 4.285 \times 10^6 \text{ lbf} \text{ (on each side)} \]

Therefore, the net downward vertical force is
\[ F_y = F_y + 2W = 2.596 \times 10^8 + 2 \times 0.04285 \times 10^8 = 2.64 \times 10^8 \text{ lbf} \]

This is also the net force acting on the tunnel since the horizontal forces acting on the right and left side of the tunnel cancel each other since they are equal and opposite.

Discussion  The weight of the two water blocks on the sides represents only about 3.3% of the total vertical force on the tunnel. Therefore, to obtain a reasonable first approximation for deep tunnels, these volumes can be neglected, yielding \( F_y = 2.596 \times 10^8 \text{ lbf} \). A more conservative approximation would be to estimate the force on the bottom of the lake if the tunnel were not there. This yields \( F_y = 2.995 \times 10^8 \text{ lbf} \). The actual force is between these two estimates, as expected.
Solution A hemispherical dome on a level surface filled with water is to be lifted by attaching a long tube to the top and filling it with water. The required height of water in the tube to lift the dome is to be determined.

Assumptions 1 Atmospheric pressure acts on both sides of the dome, and thus it can be ignored in calculations for convenience. 2 The weight of the tube and the water in it is negligible.

Properties We take the density of water to be 1000 kg/m³ throughout.

Analysis We take the dome and the water in it as the system. When the dome is about to rise, the reaction force between the dome and the ground becomes zero. Then the free body diagram of this system involves the weights of the dome and the water, balanced by the hydrostatic pressure force from below. Setting these forces equal to each other gives

$$ \sum F_y = 0 : \quad F_y = W_{dome} + W_{water} $$

$$ \rho g (h + R) \pi R^2 = m_{dome} g + m_{water} g $$

Solving for \( h \) gives

$$ h = \frac{m_{dome} + m_{water}}{\rho \pi R^2} - R = \frac{m_{dome} + \rho \left[\frac{4\pi R^3}{6}\right]}{\rho \pi R^2} - R $$

Substituting,

$$ h = \frac{(50,000 \text{ kg}) + 4\pi (1000 \text{ kg/m}^3)(3 \text{ m})^3 / 6}{(1000 \text{ kg/m}^3)\pi (3 \text{ m})^2} - (3 \text{ m}) = 0.76839 \text{ m} \approx 0.77 \text{ m} $$

Therefore, this dome can be lifted by attaching a tube which is about 77 cm long.

Discussion Note that the water pressure in the dome can be changed greatly by a small amount of water in the vertical tube. Two significant digits in the answer is sufficient for this problem.
Solution

The water in a reservoir is restrained by a triangular wall. The total force (hydrostatic + atmospheric) acting on the inner surface of the wall and the horizontal component of this force are to be determined.

Assumptions

1. Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.
2. Friction at the hinge is negligible.

Properties

We take the density of water to be 1000 kg/m³ throughout.

Analysis

The length of the wall surface underwater is

\[ b = \frac{25 \text{ m}}{\sin 60^\circ} = 28.87 \text{ m} \]

The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic force on the surface,

\[ F_R = P_{avg}A = (P_{avg} + \rho gh_c)A \]

\[ = \left[ 100,000 \text{ N/m}^2 + \left(1000 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)(12.5 \text{ m}) \right] \left(150 \times 28.87 \text{ m}^2\right) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) \]

\[ = 9.64 \times 10^8 \text{ N} \]

Noting that

\[ \frac{P_o}{\rho g \sin 60^\circ} = \frac{100,000 \text{ N/m}^2}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \sin 60^\circ} \left(\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}}\right) = 11.77 \text{ m} \]

the distance of the pressure center from the free surface of water along the wall surface is

\[ y_p = y + \frac{b}{2} + \frac{b^2}{12 \left(\frac{P_o}{\rho g \sin \theta} + \frac{b}{2} \right)} = 28.87 \text{ m} + \frac{(28.87 \text{ m})^2}{12 \left(0 + \frac{28.87 \text{ m}}{2} + 11.77 \text{ m}\right)} = 17.1 \text{ m} \]

The magnitude of the horizontal component of the hydrostatic force is simply \( F_R \sin \theta \),

\[ F_H = F_R \sin \theta = (9.64 \times 10^8 \text{ N})\sin 60^\circ = 8.35 \times 10^8 \text{ N} \]

Discussion

Atmospheric pressure is usually ignored in the analysis for convenience since it acts on both sides of the walls.
Solution A U-tube that contains water in its right arm and another liquid in its left arm is rotated about an axis closer to the left arm. For a known rotation rate at which the liquid levels in both arms are the same, the density of the fluid in the left arm is to be determined.

Assumptions 1 Both the fluid and the water are incompressible fluids. 2 The two fluids meet at the axis of rotation, and thus there is only water to the right of the axis of rotation.

Properties We take the density of water to be 1000 kg/m³.

Analysis The pressure difference between two points 1 and 2 in an incompressible fluid rotating in rigid body motion (the same fluid) is given by

\[ P_2 - P_1 = \frac{\rho \omega^2}{2} (r_2^2 - r_1^2) - \rho g (z_2 - z_1) \]

where

\( \omega = 2 \pi \frac{50 \text{ rev/min}}{60 \text{ s}} = 5.236 \text{ rad/s} \)

(for both arms of the U-tube).

The pressure at point 2 is the same for both fluids, so are the pressures at points 1 and \( 1^* \) (\( P_1 = P_1^* = P_{\text{am}} \)). Therefore, \( P_2 - P_1 \) is the same for both fluids. Noting that \( z_2 - z_1 = -h \) for both fluids and expressing \( P_2 - P_1 \) for each fluid,

Water:

\[ P_2 - P_1^* = \frac{\rho_w \omega^2}{2} (0 - R_2^2) - \rho_w g(-h) = \rho_w (-\omega^2 R_2^2 / 2 + gh) \]

Fluid:

\[ P_2 - P_1 = \frac{\rho_f \omega^2}{2} (0 - R_1^2) - \rho_f g(-h) = \rho_f (-\omega^2 R_1^2 / 2 + gh) \]

Setting them equal to each other and solving for \( \rho_f \) gives

\[ \rho_f = \frac{-\omega^2 R_2^2 / 2 + gh}{-\omega^2 R_1^2 / 2 + gh} P_w = \frac{-\omega^2 R_2^2 / 2 + gh}{-\omega^2 R_1^2 / 2 + gh} (\text{1000 kg/m}^3) = \text{677 kg/m}^3 \]

Discussion Note that this device can be used to determine relative densities, though it wouldn’t be very practical.
Solution  A vertical cylindrical tank is completely filled with gasoline, and the tank is rotated about its vertical axis at a specified rate while being accelerated upward. The pressures difference between the centers of the bottom and top surfaces, and the pressures difference between the center and the edge of the bottom surface are to be determined.

Assumptions  1 The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body.  2 Gasoline is an incompressible substance.

Properties  The density of the gasoline is given to be 740 kg/m³.

Analysis  The pressure difference between two points 1 and 2 in an incompressible fluid rotating in rigid body motion is given by

\[ P_2 - P_1 = \frac{\rho \omega^2}{2} (r_2^2 - r_1^2) - \rho (g + a_z)(z_2 - z_1). \]

The effect of linear acceleration in the vertical direction is accounted for by replacing \( g \) by \( g + a_z \). Then,

\[ P_2 - P_1 = \frac{\rho \omega^2}{2} (r_2^2 - r_1^2) - \rho (g + a_z)(z_2 - z_1) \]

where \( R = 0.50 \text{ m} \) is the radius, and

\[ \omega = 2\pi n = 2\pi (90 \text{ rev/min}) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 9.425 \text{ rad/s} \]

(a) Taking points 1 and 2 to be the centers of the bottom and top surfaces, respectively, we have \( r_1 = r_2 = 0 \) and \( z_2 - z_1 = h = 3 \text{ m} \). Then,

\[ P_{\text{center, top}} - P_{\text{center, bottom}} = 0 - \rho (g + a_z)(z_2 - z_1) = -\rho (g + a_z)h \]

\[ = -(740 \text{ kg/m}^3)(9.81 \text{ m/s}^2 + 5)(2 \text{ m}) \left( \frac{1 \text{kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 21.8 \text{kN/m}^2 = 21.9 \text{ kPa} \]

(b) Taking points 1 and 2 to be the center and edge of the bottom surface, respectively, we have \( r_1 = 0 \), \( r_2 = R \), and \( z_2 = z_1 = 0 \). Then,

\[ P_{\text{edge, bottom}} - P_{\text{center, bottom}} = \frac{\rho \omega^2}{2} (R_2^2 - 0) - 0 = \frac{\rho \omega^2 R^2}{2} \]

\[ = (740 \text{ kg/m}^3)(9.425 \text{ rad/s})^2 (0.50 \text{ m})^2 \left( \frac{1 \text{kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 8.22 \text{kN/m}^2 = 8.22 \text{ kPa} \]

Discussion  Note that the rotation of the tank does not affect the pressure difference along the axis of the tank. Likewise, the vertical acceleration does not affect the pressure difference between the edge and the center of the bottom surface (or any other horizontal plane).
Solution  A rectangular water tank open to the atmosphere is accelerated to the right on a level surface at a specified rate. The maximum pressure in the tank above the atmospheric level is to be determined.

**Assumptions**  1 The road is horizontal during acceleration so that acceleration has no vertical component \((a_z = 0)\). 2 Effects of splashing, breaking and driving over bumps are assumed to be secondary, and are not considered. 3 The vent is never blocked, and thus the minimum pressure is the atmospheric pressure.

**Properties**  We take the density of water to be 1000 kg/m\(^3\).

**Analysis**  We take the \(x\)-axis to be the direction of motion, the \(z\)-axis to be the upward vertical direction. The tangent of the angle the free surface makes with the horizontal is

\[
\tan \theta = \frac{a_x}{g + a_z} = \frac{2}{9.81 + 0} = 0.2039 \quad \text{(and thus } \theta = 11.5^\circ)\]

The maximum vertical rise of the free surface occurs at the back of the tank, and the vertical midsection experiences no rise or drop during acceleration. Then the maximum vertical rise at the back of the tank relative to the neutral midplane is

\[
\Delta z_{\text{max}} = \left( \frac{L}{2} \right) \tan \theta = \left( \frac{5 \text{ m}}{2} \right) \times 0.2039 = 0.510 \text{ m}
\]

which is less than 1.5 m high air space. Therefore, water never reaches the ceiling, and the maximum water height and the corresponding maximum pressure are

\[
h_{\text{max}} = h_0 + \Delta z_{\text{max}} = 2.50 + 0.510 = 3.01 \text{ m}
\]

\[
P_{\text{max}} = P_t = \rho gh_{\text{max}} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3.01 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 29.5 \text{ kN/m}^2 = 29.5 \text{ kPa}
\]

**Discussion**  It can be shown that the gage pressure at the bottom of the tank varies from 29.5 kPa at the back of the tank to 24.5 kPa at the midsection and 19.5 kPa at the front of the tank.
Solution  The previous problem is reconsidered. The effect of acceleration on the slope of the free surface of water in the tank as the acceleration varies from 0 to 5 m/s² in increments of 0.5 m/s² is to be investigated.

Analysis  The EES Equations window is printed below, followed by the tabulated and plotted results.

```
"a_x=5 [m/s^2]"
g=9.81 [m/s^2]
rho=1000 [kg/m^3]
L=5 [m]
h0=2.5 [m]
a_z=0 [m/s^2]
tan(theta)=a_x/(g+a_z)
h_max=h0+(L/2)*tan(theta)
P_max=rho*g*h_max*Convert(Pa, kPa)
```

<table>
<thead>
<tr>
<th>Acceleration (a_x), m/s²</th>
<th>Free surface angle, (\theta)</th>
<th>Maximum height (h_{max}), m</th>
<th>Maximum pressure (P_{max}), kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2.5</td>
<td>24.53</td>
</tr>
<tr>
<td>1</td>
<td>5.82</td>
<td>2.755</td>
<td>27.02</td>
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<td>3.01</td>
<td>29.52</td>
</tr>
<tr>
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<td>17</td>
<td>3.265</td>
<td>32.02</td>
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<tr>
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<td>22.18</td>
<td>3.519</td>
<td>34.52</td>
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<td>37.02</td>
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<td>7</td>
<td>35.51</td>
<td>4.284</td>
<td>42.02</td>
</tr>
<tr>
<td>8</td>
<td>39.2</td>
<td>4.539</td>
<td>44.52</td>
</tr>
<tr>
<td>9</td>
<td>42.53</td>
<td>4.794</td>
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<td>10</td>
<td>45.55</td>
<td>5.048</td>
<td>49.52</td>
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<td>11</td>
<td>48.27</td>
<td>5.303</td>
<td>52.02</td>
</tr>
<tr>
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<td>50.73</td>
<td>5.558</td>
<td>54.52</td>
</tr>
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<td>52.96</td>
<td>5.813</td>
<td>57.02</td>
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<td>14</td>
<td>54.98</td>
<td>6.068</td>
<td>59.52</td>
</tr>
<tr>
<td>15</td>
<td>56.82</td>
<td>6.323</td>
<td>62.02</td>
</tr>
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</table>
**Discussion**  
Note that water never reaches the ceiling, and a full free surface is formed in the tank.
**Solution**  An elastic air balloon submerged in water is attached to the base of the tank. The change in the tension force of the cable is to be determined when the tank pressure is increased and the balloon diameter is decreased in accordance with the relation $P = CD^{-2}$.

**Assumptions**  1 Atmospheric pressure acts on all surfaces, and thus it can be ignored in calculations for convenience. 2 Water is an incompressible fluid. 3 The weight of the balloon and the air in it is negligible.

**Properties**  We take the density of water to be 1000 kg/m$^3$.

**Analysis**  The tension force on the cable holding the balloon is determined from a force balance on the balloon to be

$$F_{cable} = F_B - W_{balloon} = F_B$$

The buoyancy force acting on the balloon initially is

$$F_{B,1} = \rho_w g V_{balloon,1} = \rho_w g \frac{\pi D_1^3}{6} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \frac{\pi(0.30 \text{ m})^3}{6} = 138.7 \text{ N}$$

The variation of pressure with diameter is given as $P = CD^{-2}$, which is equivalent to $D = \sqrt{C/P}$. Then the final diameter of the ball becomes

$$D_2 = \sqrt{\frac{C}{P_2}} = \sqrt{\frac{P_1}{P_2}} \rightarrow D_2 = D_1 \sqrt{\frac{P_1}{P_1}} = 0.075 \text{ m}$$

The buoyancy force acting on the balloon in this case is

$$F_{B,2} = \rho_w g V_{balloon,2} = \rho_w g \frac{\pi D_2^3}{6} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \frac{\pi(0.075 \text{ m})^3}{6} = 2.2 \text{ N}$$

Then the percent change in the cable for becomes

$$\text{Change\%} = \frac{F_{cable,1} - F_{cable,2}}{F_{cable,1}} \times 100 = \frac{F_{B,1} - F_{B,2}}{F_{B,1}} \times 100 = \frac{138.7 - 2.2}{138.7} \times 100 = 98.4\%.$$ 

Therefore, increasing the tank pressure in this case results in 98.4% reduction in cable tension.

**Discussion**  We can obtain a relation for the change in cable tension as follows:

$$\text{Change\%} = \frac{F_{B,1} - F_{B,2}}{F_{B,1}} \times 100 = \frac{\rho_w g V_{balloon,1} - \rho_w g V_{balloon,2}}{\rho_w g V_{balloon,1}} \times 100$$

$$= 100 \left(1 - \frac{V_{balloon,2}}{V_{balloon,1}}\right) = 100 \left(1 - \frac{D_2^3}{D_1^3}\right) = 100 \left(1 - \left(\frac{P_1}{P_2}\right)^{3/2}\right).$$
Solution  The previous problem is reconsidered. The effect of the air pressure above the water on the cable force as the pressure varies from 0.1 MPa to 10 MPa is to be investigated.

Analysis  The EES Equations window is printed below, followed by the tabulated and plotted results.

\[
P_1=0.1 \text{ "MPa"}
\]

\[
\text{Change}=100\times(1-(P_1/P_2)^{1.5})
\]

<table>
<thead>
<tr>
<th>Tank pressure $P_2$, MPa</th>
<th>%Change in cable tension</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>91.06</td>
</tr>
<tr>
<td>1.467</td>
<td>98.22</td>
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<td>2.433</td>
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<td>10.17</td>
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<td>11.13</td>
<td>99.91</td>
</tr>
<tr>
<td>12.1</td>
<td>99.92</td>
</tr>
<tr>
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<td>14.03</td>
<td>99.94</td>
</tr>
<tr>
<td>15</td>
<td>99.95</td>
</tr>
</tbody>
</table>

Discussion  The change in cable tension is at first very rapid, but levels off as the balloon shrinks to nearly zero diameter at high pressure.
Solution  An iceberg floating in seawater is considered. The volume fraction of the iceberg submerged in seawater is to be determined, and the reason for their turnover is to be explained.

Assumptions  1 The buoyancy force in air is negligible. 2 The density of iceberg and seawater are uniform.

Properties  The densities of iceberg and seawater are given to be 917 kg/m³ and 1042 kg/m³, respectively.

Analysis  
(a) The weight of a body floating in a fluid is equal to the buoyant force acting on it (a consequence of vertical force balance from static equilibrium). Therefore,

\[
W = F_B
\]

\[
\rho_{\text{body}} g V_{\text{total}} = \rho_{\text{fluid}} g V_{\text{submerged}}
\]

\[
\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{fluid}}} = \frac{\rho_{\text{iceberg}}}{\rho_{\text{seawater}}} = \frac{917}{1042} = 0.880 \text{ or } 88\%
\]

Therefore, 88% of the volume of the iceberg is submerged in this case.

(b) Heat transfer to the iceberg due to the temperature difference between the seawater and an iceberg causes uneven melting of the irregularly shaped iceberg. The resulting shift in the center of mass causes the iceberg to turn over.

Discussion  The submerged fraction depends on the density of seawater, and this fraction can differ in different seas.
Design and Essay Problems

3-165
Solution  We are to discuss the design of shoes that enable people to walk on water.

Discussion  Students’ discussions should be unique and will differ from each other.

3-166
Solution  We are to discuss how to measure the volume of a rock without using any volume measurement devices.

Analysis  The volume of a rock can be determined without using any volume measurement devices as follows: We weigh the rock in the air and then in the water. The difference between the two weights is due to the buoyancy force, which is equal to \( F_B = \rho_{\text{water}} g V_{\text{body}} \). Solving this relation for \( V_{\text{body}} \) gives the volume of the rock.

Discussion  Since this is an open-ended design problem, students may come up with different, but equally valid techniques.
Chapter 3 Pressure and Fluid Statics

3-167

Solution  The maximum total weight and mass of a razor blade floating on water along with additional weights on the razor blade is to be estimated.

Assumptions  1 Surface tension acts only on the outer edges of the blade. 2 The blade is approximated as a rectangle for simplicity – three-dimensional corner effects are neglected. 3 In the limiting case, the water surface is vertical at the junction with the razor blade – as soon as the water starts to move over the razor blade surface, the razor blade would sink.

Properties  The surface tension of water at 20°C is 0.073 N/m, and its density is 998.0 kg/m³

Analysis  (a) Considering surface tension alone, the total upward force due to surface tension is the perimeter of the razor blade times the surface tension acting at contact angle \( \phi \). But here, the limiting case is when \( \phi = 180^\circ \). This must balance the weight \( W \),

\[
W = -2 \sigma_s (L + w) \cos \phi = -2 \left( \frac{0.073}{m} \right) (0.043 + 0.022) \ m \cdot \cos(180^\circ) = 0.00949 \ N
\]

which we convert to mass by dividing by the gravitational constant, namely,

\[
m = \frac{W}{g} = \frac{0.00949 \ N}{9.807 \ m/s^2} \left( \frac{\text{kg} \cdot \text{m/s}^2}{\text{N}} \right) \left( 1000 \ \text{g} \right) = 0.96768 \ \text{g}
\]

The values and properties are given to only two significant digits, so our final results are \( W = 0.0095 \ N \) and \( m = 0.97 \ \text{g} \).

(b) Since the razor blade pushes down on the water, the pressure at the bottom of the blade is larger than that at the top of the blade due to hydrostatic effects as sketched. Thus, more weight can be supported due to the difference in pressure. Since \( P_{\text{below}} = P_{\text{atm}} + \rho g h \), we write

\[
W = -2 \sigma_s (L + w) \cos \phi + \rho g h L w
\]

However, from the hint, we know also that the maximum possible depth is \( h = \frac{2 \sigma_s}{\rho g} \). When we set \( \phi = 180^\circ \) and substitute this expression for \( h \), we can solve for \( W \),

\[
W = -2 \sigma_s (L + w) \cos \phi + \sqrt{2 \rho g \sigma_s} L w
\]

\[
= -2 \left( \frac{0.073}{m} \right) (0.043 + 0.022) \ m \cdot \cos(180^\circ)
+ \sqrt{2 \left( 998.0 \ \text{kg/m}^3 \right) \left( 9.807 \ \text{m/s}^2 \right) \left( \frac{0.073}{m} \right) \left( \frac{\text{N}}{\text{kg} \cdot \text{m/s}^2} \right) (0.043 \ m) (0.022 \ m)}
\]

\[
= 0.045250 \ \text{N}
\]

Again, since the values given to only two significant digits, our final results are \( W = 0.0495 \ N \) and \( m = 4.6 \ \text{g} \).

Discussion  The hydrostatic pressure component has greatly increased the amount of weight that can be supported, by a factor of almost 5.