# Løsningsforslag kontinuasjonseksamen 25. Feb 2005

FO 005A og 001A Matematiske metoder 1 og Matematikk 1.

1) $y = \dfrac{x^2}{\sqrt[3]{x}} = x^2 \cdot x^{-\frac{1}{3}} = x^{\frac{5}{3}}$

$y' = \frac{5}{3} x^{2/3} = \underline{\underline{\frac{5}{3}\sqrt[3]{x^2}}}$

2) $y = \tan(2x) \cdot \cos^2 x$

$y' = 2\dfrac{1}{\cos^2 2x} \cdot \cos^2 x - 2\sin x \cdot \cos x \cdot \tan 2x$

$= \underline{\underline{2\left(\dfrac{\cos x}{\cos 2x}\right)^2 - \dfrac{\sin^2 2x}{\cos 2x}}}$

b) $y^2 = \ln(x+y) + \frac{2}{x}$

Hvis likningen er oppfylt når vi setter inn $(x, y) = (2, -1)$ ligger punktet på kurven $K$.

VS: $1$ HS: $\ln(1) + \frac{2}{2} = 1$

$\Rightarrow$ VS = HS $\Rightarrow$ punktet ligger på kurven.

Implisitt derivasjon gir:

$2yy' = \dfrac{1+y'}{x+y} - \dfrac{2}{x^2}$

Innsatt $(x, y) = (2, -1)$

$$2yy' = \frac{1+y'}{x+y} - \frac{2}{x^2}$$

$$-2y' = (1+y')/1 - \frac{1}{2}$$

$$-3y' = \frac{1}{2}$$

$$y' = -\frac{1}{6}$$

Tangent likningen blir

$$(y+1) \quad \frac{1}{6} = - \quad (x-2) \Rightarrow \underline{\underline{y = -\frac{x}{6} - \frac{2}{3}}}$$

Oppgave 2

$$\int \sqrt{1+3x}\, dx \qquad \begin{aligned} u &= 1+3x \\ du &= 3\,dx \end{aligned}$$

$$\underline{\underline{= \frac{2}{9}(1+3x)^{3/2} + C}}$$

2) $\int x \tan^2(x^2+1)\, dx$

$$u = x^2+1$$

$$du = 2x\, dx$$

$$\frac{1}{2}\int \tan^2 u\, du$$

$$= -\frac{1}{2}\left[u - \tan u\right] + c$$

$$\underline{\underline{= -\frac{1}{2}(x^2+1) + \frac{1}{2}\tan(x^2+1) + c}}$$

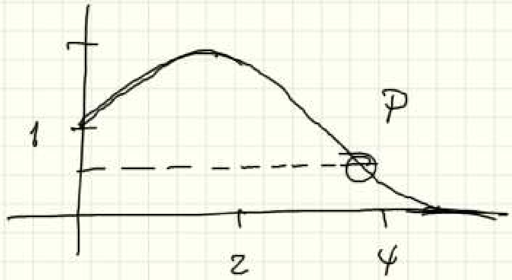
## Oppgave 3

a) $f(x) = 1 + \sin x \qquad 0 \leq x \leq \frac{3\pi}{2}$

$1 + \sin x = \frac{1}{2}$

$\Rightarrow \sin x = -\frac{1}{2}$

$x = \pi + \frac{\pi}{6} = \frac{7}{6}\pi$




x koordinaten blir $\underline{\underline{\frac{7}{6}\pi}}$

b) Rotasjon av F om x-aksen.
Velger å bruke skive metoden.

$dV = \pi r^2 \, dx \qquad r = 1 + \sin x$

$$V = \int_{x=0}^{x=\frac{3}{2}\pi} dV = \pi \int_0^{\frac{3}{2}\pi} 1 + 2\sin x + \sin^2 x \, dx$$

$$V = \pi \left[ \frac{3}{2}x - 2\cos x - \frac{1}{4}\sin 2x \right]_0^{3/2\,\pi}$$

$$\underline{\underline{= \frac{9}{4}\pi^2 + 2\pi}}$$

c) Simpsons metode til å finne buelengden.

Vi skal finne integralet

$$s = \int_0^{\pi} \sqrt{1 + y'^2} \, dx \qquad y' = \cos x$$

$$S = \int_0^{\pi} \sqrt{1+\cos^2 x}\, dx$$ 6 intervall.

$$y = \sqrt{1+\cos^2 x}$$

| $i$ | $x_i$ | $y_i$ |
|---|---|---|
| 0 | 0 | $\sqrt{2}$ |
| 1 | $\pi/6$ | $\frac{\sqrt{7}}{2}$ |
| 2 | $\pi/3$ | $\frac{\sqrt{5}}{2}$ |
| 3 | $\pi/2$ | 1 |
| 4 | $\frac{2}{3}\pi$ | $\frac{\sqrt{5}}{2}$ |
| 5 | $5/6\,\pi$ | $\frac{\sqrt{7}}{2}$ |
| 6 | $\pi$ | $\sqrt{2}$ |

$$S = \frac{\pi}{3\cdot 6}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6\right)$$

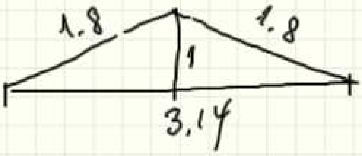
$$= \frac{\pi}{9}\left(\sqrt{2} + 2\sqrt{7} + \sqrt{5} + 2\right)$$

$$\underline{\underline{= \ 3.82}}$$

virker fornuftig

Kontroll:

Ser på figur og ser om det er rimlig




$$S \gtrsim \sqrt{9+4} \approx 3.6$$

c) Ved rotasjon om $x = -2$

Velger å bruke sylinderskall metoden

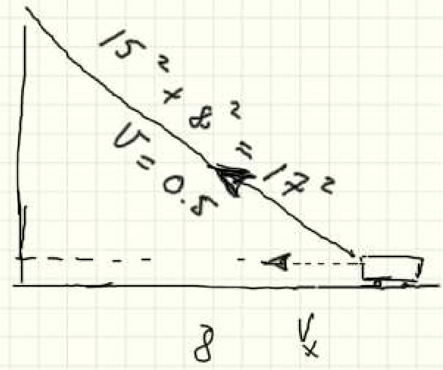
$$dV = 2\pi r \cdot h \cdot dx \qquad r = 2 + x$$

$$V = 2\pi \int_{x=0}^{x=3/2\,\pi} (2+x)(1+\sin x)\, dx$$

$$= 2\pi \left[2x + \frac{x^2}{2} - 2\cos x + \sin x - x\cos x\right]_0^{3/2\,\pi}$$

$$\underline{\underline{= 2\pi\left[\frac{9}{8}\pi^2 + 3\pi + 1\right]}}$$

## Oppgave 4




$v_x = v \cdot \cos\varphi$

$\cos\varphi = \frac{8}{17}$

$v_x = 0,5 \cdot \frac{8}{17}$ m/s

$= \frac{4}{17}$ m/s

## Oppgave 5

$P(t)$

$$dP = k P^2 + 1000$$

$$\frac{dP}{dt} = \frac{P^2}{100 \cdot 10^3} + 10^3$$

$$\frac{dP}{P^2 + 10^8} = 10^{-5} \cdot dt$$

$$\frac{\arctan \frac{P}{10^4}}{10^4} = 10^{-5} t + C$$

$$P(t) = 10^4 \tan\left(\frac{t}{10} + C\right)$$

$$P(t=0) = 10^4 \cdot \tan C = 1000$$

$$\Rightarrow \tan C = \frac{1}{10} \Rightarrow C = \arctan\frac{1}{10}$$

$$\underline{\underline{P(t) = 10^4 \tan\left(\frac{t}{10} + \arctan\frac{1}{10}\right)}}$$ $\arctan\frac{1}{10} \approx 0.1$